MATHEMATICS PAPER IIB

COORDINATE GEOMETRY AND CALCULUS.

Time: 3hrs Max. Marks.75

Note: This question paper consists of three sections A, B and C.

SECTION -A

Very Short Answer Type Questions. 10 X 2 = 20

1. Find the condition that the tangents Drawn from (0,0) to $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ be perpendicular to each other.

2. Find the length of the chord intercepted by the circle $x^2 + y^2 - x + 3y - 22 = 0$ on the line $y = x - 3$

3. Show that $x^2 + y^2 + 2lx + 4 = 0$; $x^2 + y^2 + 2my - g = 0$ Circles intersect each other orthogonally.

4. Find the vertex and focus of $x^2 - 6x - 6y + 6 = 0$.

5. If $3x - 4y + k = 0$ is a tangent to $x^2 - 4y^2 = 5$, find value of k.

6. Evaluate $\int e^x \left(\frac{1 + x\log x}{x}\right) dx$ on $(0, \infty)$.

7. Evaluate $\int \frac{dx}{1+e^x}, x \in R$

8. Evaluate $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x} dx$

9. Evaluate $\int_0^{\pi/4} \sec^4 \theta \, d\theta$

10. Solve $\frac{dy}{dx}=\frac{2x-y+1}{x+2y-3}$

SECTION -B

Short Answer Type Questions.

Answer Any Five of the Following 5 X 4 = 20

11. If the two circles $x^2 + y^2 + 2gx + 2fy = 0$ and $x^2 + y^2 + 2g'x + 2f'y = 0$ touch each other then show that $f'g = fg'$.

12. Find the equations of the tangents to the circle $x^2+y^2 + 2x - 2y - 3 = 0$ which are perpendicular to $3x - y+4=0$

13. Find the condition for the line $lx + my + n = 0$ to be a normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

14. Find the equation of the tangents to $9x^2 + 16y^2 = 144$ which makes equal intercepts on coordinate axes.

15. If e, e_1 be the eccentricity of a hyperbola and its conjugate hyperbola then $\frac{1}{e^2}+\frac{1}{e_1^2}=1$.

16. Evaluate $\lim_{n\to\infty}\left[\frac{(n!)^{1/n}}{n}\right]$

17. Solve $(1+x^2)\frac{dy}{dx}+y=e^{\tan^{-1}x}$

SECTION - C

Long Answer Type Questions.

Answer Any Five of the Following 5 X 7 = 35

18. Find the equation of the circle passing through (0,0) and making intercepts 4,3 on X – axis and Y –axis respectively.

19. Show that the equation to the pair of tangents to the circle S = 0 from $P(x_1, y_1)$ is $S_1^2 = S_{11}S$.

20. Prove that the tangents at the extremities of a focal chord of a parabola intersect at right angles on the directrix.

21. $\int (6x+5)\sqrt{6-2x^2+x}\, dx$

22. Evaluate $\int \frac{1}{1+x^4} dx$

23. Find the area bounded between the curves $y^2 = 4ax,\ x^2 = 4by\left(a > 0, b > 0\right)$.

24. Solve $(y^2 - 2xy)dx + (2xy - x^2)dy = 0$

Maths 2B Paper 3 - Solutions

1. Find the condition that the tangents Drawn from (0,0) to S ≡ $x^2 + y^2 + 2gx + 2fy + c = 0$ be perpendicular to each other.

Sol. Let θ be the angle between the pair of

Tangents then $\tan\frac{\theta}{2} = \frac{r}{\sqrt{S_{11}}}$

$$given \ \ \theta = \frac{\pi}{2}, \text{ radius } r = \sqrt{g^2 + f^2 - c}$$

$$S_{11} = x\frac{2}{1} + y\frac{2}{1} + 2gx_1 + 2fy_1 + c = 0 + c = c$$

$$\tan 45^0 = \frac{\sqrt{g^2 + f^2 - c}}{\sqrt{x\frac{2}{1} + y\frac{2}{1} + 2gx_1 + 2fy_1 + c}}$$

$$\Rightarrow 1 = \frac{\sqrt{g^2 + f^2 - c}}{\sqrt{0+0+0+0+c}}$$

$$\Rightarrow g^2 + f^2 - c = c$$

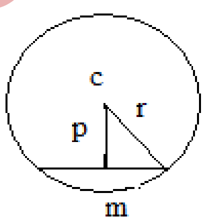
2. Find the length of the chord intercepted by the circle $x^2 + y^2 - x + 3y - 22 = 0$ on the line $y = x - 3$

Sol. Equation of the circle is $S \equiv x^2 + y^2 - x + 3y - 22 = 0$. Center C $\left(\frac{1}{2}, -\frac{3}{2}\right)$ and

Radius $r = \sqrt{g^2 + f^2 - c} = \sqrt{\frac{1}{4} + \frac{9}{4} + 22} = \sqrt{\frac{1+9+88}{4}} = \sqrt{\frac{98}{44}}$

Equation of the line is $y = x - 3 \Rightarrow x - y - 3 = 0$

P = distance from the centre to the line



$= \frac{\left|\frac{1}{2} + \frac{3}{2} - 3\right|}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$

Length of the chord $= 2\sqrt{r^2 - p^2} = 2\sqrt{\frac{98}{4} - \frac{1}{2}} = 2\sqrt{\frac{98-2}{2}} = \sqrt{96} = 4\sqrt{6}$ units

3. Show that $x^2 + y^2 + 2lx + 4 = 0$; $x^2 + y^2 + 2my - g = 0$ circles intersect each other orthogonally.

Sol. Given circles $x^2 + y^2 + 2lx + 4 = 0$; $x^2 + y^2 + 2my - g = 0$ from these equations,

$g_1 = -l;\ f_1 = 0,\ c_1 = g,\ g_2 = 0,\ f_2 = m,\ c_2 = -g$

Now $2g_1g_2 + 2f_1f_2 = c_1 + c_2$

$2(-l)(0) + 2(0)(m) = g - g$

$0 = 0$ ∴Two circles are orthogonal.

4. Find the vertex and focus of $x^2 - 6x - 6y + 6 = 0$.

Sol. Given parabola is

$x^2 - 6x - 6y + 6 = 0$

$x^2 - 6x = 6y - 6$

$(x - 3)^2 - 9 = 6y - 6$

$(x - 3)^2 = 6y + 3$

$(x - 3)^2 = 6\left(y + \frac{1}{2}\right) = 6\left[y - \left(\frac{-1}{2}\right)\right]$

$\therefore h = 3,\ k = \frac{-1}{2},\ a = \frac{6}{4} = \frac{3}{2}$

Vertex = (h, k) = $\left(3, \frac{-1}{2}\right)$

Focus = (h, k+a) = $\left(3, -\frac{1}{2} + \frac{3}{2}\right) = (3,1)$

5. If $3x - 4y + k = 0$ is a tangent to $x^2 - 4y^2 = 5$, find value of k.

Sol. Equation of the hyperbola $x^2 - 4y^2 = 5$

$\frac{x^2}{5} - \frac{y^2}{(5/4)} = 1 \Rightarrow a^2 = 5, b^2 = \frac{5}{4}$

Equation of the line is 3x – 4y + k = 0

$4y = 3x + k \Rightarrow y = \frac{3}{4}x + \frac{k}{4}$ ---- (1)

$m = \frac{3}{4},\ c = \frac{k}{4}$

If (1) is a tangent to the hyperbola then

$c^2 = a^2m^2 - b^2$

$\Rightarrow \frac{k^2}{16} = 5\cdot\frac{9}{16} - \frac{5}{4}$

$\Rightarrow k^2 = 45 - 20 = 25 \Rightarrow k = \pm 5$

6. Evaluate $\int e^x\left(\frac{1+x\log x}{x}\right)dx$ **on (0, ∞).**

Sol. $\int e^x\left(\frac{1+x\log x}{x}\right)dx = \int e^x\left(\log x + \frac{1}{x}\right)dx$

$= e^x \log x + C$

7. Evaluate $\int \frac{dx}{1+e^x}, x \in R$

Sol. $\int \frac{dx}{1+e^x} = \int\left(\frac{1+e^x-e^x}{1+e^x}\right)dx$

$= \int\left(1 - \frac{e^x}{1+e^x}\right)dx = x - \log(1+e^x) + C$

8. Evaluate $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x}dx$

Sol. Let $I = \int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x}dx$ ... (i)

$$I = \int_{-\pi/2}^{\pi/2} \frac{\cos(\pi/2 - \pi/2 - x)dx}{1+e^{-x}} \left(\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx \right)$$

Adding (1) and (2),

$$= \int_{-\pi/2}^{\pi/2} \frac{e^x \cos x dx}{1+e^x} ----(2)$$

$$2I = \int_{-\pi/2}^{\pi/2} \frac{\cos x(1+e^x)}{1+e^x} dx = \int_{-\pi/2}^{\pi/2} \cos x dx$$

$$2I = 2\int_0^{\pi/2} \cos x\, dx \left(\because \cos x \text{ is even function}\right)$$

$$\Rightarrow I = [\sin x]_0^{\pi/2} \Rightarrow I = 1$$

9. Evaluate $\int_0^{\pi/4} \sec^4 \theta d\theta$

$$\int_0^{\pi/4} \sec^4 \theta d\theta = \int_0^{\pi/4} \sec^2 \theta . \sec^2 \theta d\theta = \int_0^{\pi/4} \sec^2 \theta(1+\tan^2 \theta)d\theta$$

Sol. Let $= \int_0^{\pi/4} \left(\sec^2 \theta + \sec^2 \theta \tan^2 \theta\right) d\theta = \int_0^{\pi/4} \sec^2 \theta d\theta + \int_0^{\pi/4} \tan^2 \theta \sec^2 \theta d\theta$

$$= \tan \theta)_o^{\pi/4} + \left(\frac{\tan^3 \theta}{3}\right)_o^{\pi/4} = 1 - 0 + \frac{1}{3}(1-0) = \frac{4}{3}$$

10. Solve $\frac{dy}{dx} = \frac{2x - y + 1}{x + 2y - 3}$

Sol. b = –1, a′ = 1 ⇒ b = –a′

(x + 2y – 3)dy = (2x – y + 1)dx

(x + 2y – 3)dy – (2x – y + 1)dx = 0

(xdy + y dx) + 2y dy – 3 dy – 2x dx – dx = 0

Integrating :

$xy + y^2 - x^2 - 3x - x = c$

11. If the two circles $x^2 + y^2 + 2gx + 2fy = 0$ and $x^2 + y^2 + 2g'x + 2f'y = 0$ touch each other then show that $f'g = fg'$.

Sol. $S = x^2 + y^2 + 2gx + 2fy = 0$

Centre $C_1 = (-g, -f)$, radius $r_1 = \sqrt{g^2 + f^2}$

$S^1 = x^2 + y^2 + 2g'x + 2f'y = 0$

$C_2 = (-g', -f')$, $r_2 = \sqrt{g'^2 + f'^2}$

Given circles are touching circles,

$\therefore\ C_1C_2 = r_1 + r_2$

$\Rightarrow\ (C_1C_2)^2 = (r_1 + r_2)^2$

$$(g' - g)^2 + (f' - f)^2 = g^2 + f^2 + g'^2 + f'^2 + 2\sqrt{g^2 + f^2}\sqrt{g'^2 + f'^2}$$

$$-2(gg' + ff') = 2\{g^2g'^2 + f^2f'^2 + g^2f'^2 + f^2g'^2\}^{1/2}$$

$$\Rightarrow (gg' + ff')^2 = g^2g'^2 + f^2f'^2 + g^2f'^2 + f^2g'^2$$

$$g^2g'^2 + f^2f'^2 + 2gg'ff' = g^2g'^2 + f^2f'^2 + g^2f'^2 + f^2g'^2$$

$$\Rightarrow 2gg'ff' = g^2f'^2 + f^2g'^2$$

$$\Rightarrow g^2f'^2 + f^2g'^2 - 2gg'ff' = 0$$

$$\Rightarrow (gf' - fg')^2 = 0 \Rightarrow gf' = fg'$$

12. Find the equations of the tangents to the circle $x^2+y^2 + 2x - 2y - 3 = 0$ which are perpendicular to $3x - y + 4 = 0$

Sol. S $= x^2+y^2 + 2x-2y-3=0$, centre C(-1, 1)

and radius $r = \sqrt{1+1+3} = \sqrt{5}$

Equation of the line perpendicular to 3x-y+4 =0 is

x+ 3y +k = 0

$$\sqrt{5} = \left|\frac{-1+3+k}{\sqrt{1+9}}\right| \Rightarrow 5 = \frac{(k+2)^2}{10}$$

$\Rightarrow 50 = k^2+4k+4 \quad \Rightarrow k^2+4k-46 = 0$

$$\Rightarrow k = \frac{-4 \pm \sqrt{16+184}}{2}$$

$$k = \frac{-4 \pm 10\sqrt{2}}{2} = -2 \pm 5\sqrt{2}$$

Equation of the required tangent is

$x + 3y - 2 \pm 5\sqrt{2} = 0$

13. Find the condition for the line lx + my + n = 0 to be a normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Sol. Equation of the ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Let lx + my + n = 0 be normal at P(a)

Equation of the normal at P(a) is :

$$\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2 \quad \dots (1)$$

Lx + my = –n ... (2)

Comparing (1) and (2)

$$\frac{l}{\left(\frac{a}{\cos\theta}\right)} = \frac{m}{\left(\frac{-b}{\sin\theta}\right)} = \frac{n}{a^2-b^2}$$

$$\frac{l\cos\theta}{a} = \frac{-m\sin\theta}{b} = \frac{-n}{a^2-b^2}$$

$$\cos\theta = \frac{-an}{l(a^2-b^2)}, \sin\theta = \frac{bn}{m(a^2-b^2)}$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\frac{a^2n^2}{l^2(a^2-b^2)^2} + \frac{b^2n^2}{m^2(a^2-b^2)^2} = 1$$

$\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2-b^2)^2}{n^2}$ is the required condition.

14. Find the equation of the tangents to 9x^2 + 16y^2 = 144 which makes equal intercepts on coordinate axes.

Sol. Equation of the ellipse is

9x^2 + 16y^2 = 144

$$\Rightarrow \frac{x^2}{16}+\frac{y^2}{9}=1$$

Equation of the tangent is $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$

Slope of the tangent = $-\frac{b\cos\theta}{a\sin\theta}=-1$

$$\cot\theta=\frac{a}{b}=\frac{4}{3}$$

$$\cos\theta=\pm\frac{4}{5},\sin\theta=\pm\frac{3}{5}$$

Equation of the tangent is:

$$\frac{x}{4}\left(\pm\frac{4}{5}\right)+\frac{y}{3}\left(\pm\frac{3}{5}\right)=1$$

$$x\pm y\pm 5=0$$

15. If e, e$_1$ be the eccentricity of a hyperbola and its conjugate hyperbola then $\frac{1}{e^2}+\frac{1}{e_1^2}=1$.

Sol. Equation of the hyperbola is

$$S=\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$$e=\sqrt{\frac{a^2+b^2}{a^2}}\Rightarrow e^2=\frac{a^2+b^2}{a^2}$$

$$\therefore \frac{1}{e^2}=\frac{a^2}{a^2+b^2} \qquad \text{...... (1)}$$

Equation of the conjugate hyperbola is

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1\Rightarrow\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$$

$$e_1=\sqrt{\frac{a^2+b^2}{b^2}}\Rightarrow e_1{}^2=\frac{a^2+b^2}{b^2}\Rightarrow\frac{1}{e_1^2}=\frac{b^2}{a^2+b^2}\quad\text{...... (2)}$$

Adding (1) and (2)

$$\frac{1}{e^2}+\frac{1}{e_1^2}=\frac{a^2}{a^2+b^2}+\frac{b^2}{a^2+b^2}=\frac{a^2+b^2}{a^2+b^2}=1$$

16. Evaluate $\lim_{n\to\infty}\left[\frac{(n!)^{1/n}}{n}\right]$

Sol: $\lim_{n\to\infty}\left[\frac{(n!)^{1/n}}{n}\right]$

$$=\lim_{n\to\infty}\left[\frac{(\underline{|n})^{1/n}}{n}\right]$$

$$=\lim_{n\to\infty}\left[\frac{(\underline{|n})}{n^n}\right]^{1/n}$$

$$\text{Let } y=\lim_{n\to\infty}\left[\frac{(\underline{|n})}{n^n}\right]^{1/n}$$

$$\log_e y=\lim_{n\to\infty}\frac{1}{n}\log\left[\left(\frac{1}{n}\right)\left(\frac{2}{n}\right)\cdots\left(\frac{i}{n}\right)\cdots\left(\frac{n}{n}\right)\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\log\frac{r}{n}$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum\log\left(\frac{r}{n}\right)$$

$$= \int_0^1 \log_e x = [x \log x]_0^1 - \int_0^1 dx$$

$$= [x \log x - x]_0^1$$

$$= [x(\log x - 1)]_0^1 = -1$$

$$\therefore y = e^{-1} = \frac{1}{e}.$$

17. Solve $(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$

Sol. $\frac{dy}{dx} + \frac{1}{1+x^2} \cdot y = \frac{e^{\tan^{-1}x}}{1+x^2}$ which linear differential equation in y.

$$\text{I.F.} = e^{\int p dx} = e^{\int \frac{dx}{1+x^2}} = e^{\tan^{-1}x}$$

Sol is $\text{y.I.F.} = \text{y.I.F} = \int \text{Q. I.F. dx}$

$$y \cdot e^{\tan^{-1}x} = \int \frac{(e^{\tan^{-1}x})^2}{1+x^2} dx \qquad \ldots (1)$$

Consider $\int \frac{(e^{\tan^{-1}x})^2}{1+x^2} dx$ put $\tan^{-1}x = t \Rightarrow \frac{dx}{1+x^2} = dt$

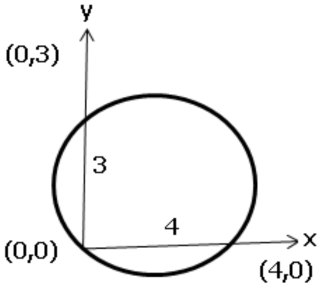
$$= \int (e^t)^2 dt = \int e^{2t} dt = \frac{e^{2t}}{2} = \frac{e^{2\tan^{-1}x}}{2}$$

Solution is $y \cdot e^{\tan^{-1}x} = \frac{e^{2\tan^{-1}x}}{2} + \frac{c}{2}$

$$2y \cdot e^{\tan^{-1}x} = e^{2\tan^{-1}x} + c$$

18. Find the equation of the circle passing through (0,0) and making intercepts 4,3 on X – axis and Y –axis respectively.

Sol.



Let the equation of the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$.

Given circle is making intercepts 4, 3 on x, y –axes respectively.

Therefore, (4,0) and (0,3) are two points on the circle.

Circle is passing through

(0,0), (4,0) and (0,3).

$(0,0) \Rightarrow 0 + 0 + 2g(0) + 2f(0) + c = 0$

$C = 0$

$(4,0) \Rightarrow 16+0+8g+2f.0+c = 0$

$G = 2$ as $c = 0$

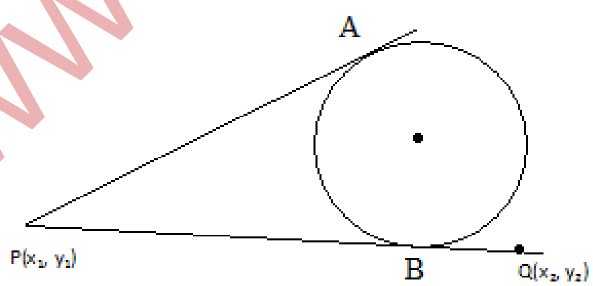
$(0,3) \Rightarrow 0+ 9+2g.0+6f +c = 0$

$f = -\frac{3}{2}$ as $c = 0$

Required equation of circle is $X^2 + y^2 -4x - 3y = 0$

19. The equation to the pair of tangents to the circle $S = 0$ from $P(x_1, y_1)$ is $S_1^2 = S_{11}S$.

Proof:



Let the tangents from P to the circle S=0 touch the circle at A and B.

Equation of AB is $S_1 = 0$.

i.e., $x_1x + y_1y + g(x + x_1) + f(y + y_1) + c = 0$ =====i)

Let Q(x_2,y_2) be any point on these tangents. Now locus of Q will be the equation of the pair of tangents drawn from P.

the line segment PQ is divided by the line AB in the ratio $-S_{11}:S_{22}$

$$\Rightarrow \frac{PB}{QB} = \left|\frac{S_{11}}{S_{22}}\right| \text{----ii)}$$

BUT $PB = \sqrt{S_{11}}, QB = \sqrt{S_{22}} \Rightarrow \frac{PB}{QB} = \frac{\sqrt{S_{11}}}{\sqrt{S_{22}}}$ ----iii)

From ii) and iii) $\Rightarrow \frac{s^2{}_{11}}{s^2{}_{22}} = \frac{S_{11}}{S_{22}}$

$\Rightarrow S_{11}S_{22} = S^2{}_{12}$

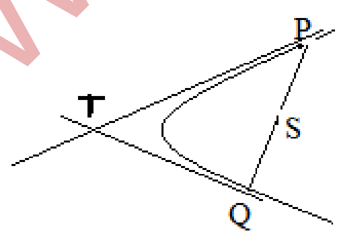
Hence locus of $Q(x_2, y_2)$ is $S_{11}S = S^2{}_1$

20. Prove that the tangents at the extremities of a focal chord of a parabola intersect at right angles on the directrix.

Sol. Let the parabola be $y^2 = 4ax$

Equation of the tangent at P(t_1) is

$t_1y = x + at_1^2$



Equation of the tangent at $Q(t_2)$ is

$$t_2 y = x + at_2^2$$

Solving, point of intersection is

$$T[at_1t_2, a(t_1 + t_2)]$$

Equation of the chord PQ is

$$(t_1 + t_2)y = 2x + 2at_1t_2$$

Since PQ is a focal chord, S (a,0) is a point on PQ.

Therefore, 0 = 2a +2a t_1t_2

$\Rightarrow$ t_1t_2 = -1.

Therefore point of intersection of the tangents is $[-a, a(t_1 + t_2)]$.

The x coordinate of this point is a constant. And that is x = -a which is the equation of the directrix of the parabola.

Hence tangents are intersecting on the directrix.

21. $\int (6x+5)\sqrt{6-2x^2+x}\,dx$

Sol.

let $6x+5=A\,\frac{d}{dx}\left(6-2x^2+x\right)+B$

$\Rightarrow$ 6x + 5 = A(1 – 4x) + B

Equating the coefficients

$$6 = -4A \Rightarrow A = \frac{-3}{2}$$

Equating the constants

A + B = 5

B = 5 – A = $5+\frac{3}{2}=\frac{13}{2}$

$$\int (6x+5)\sqrt{6-2x^2+x}\,dx$$

$$= -\frac{3}{2}\int (1-4x)\sqrt{6-2x^2+x}\,dx + \frac{13}{2}\sqrt{6-2x^2+x}\,dx$$

$$= -\frac{3}{2}\frac{(6-2x^2+x)^{3/2}}{3/2} + \frac{13}{2}\sqrt{2}\int\sqrt{3-x^2+\frac{x}{2}}dx$$

$$= -(6-2x^2+x)^{3/2} + \frac{13}{\sqrt{2}}\int\sqrt{\left(\frac{7}{4}\right)^2 - \left(x-\frac{1}{4}\right)^2}\,dx$$

$$= -(6-2x^2+x)^{3/2} + \frac{13}{\sqrt{2}}$$

$$\left(\frac{\left(x-\frac{1}{4}\right)\sqrt{3-x^2+\frac{x}{2}}}{2} + \frac{49}{32}\sin^{-1}\frac{\left(x-\frac{1}{4}\right)}{\left(\frac{7}{4}\right)}\right) + C$$

$$= -(6-2x^2+x)^{3/2} + \frac{13}{\sqrt{2}}$$

$$\left[\frac{(4x-1)\sqrt{6-2x^2+x}}{16\times 2} + \frac{49}{32}\sin^{-1}\left(\frac{4x-1}{7}\right)\right] + C$$

$$= -(6-2x^2+x)^{3/2} + \frac{13}{16}(4x-1)$$

$$\sqrt{6-2x^2+x} + \frac{637}{32\sqrt{2}}\sin^{-1}\left(\frac{4x-1}{7}\right) + C$$

22. $\int \frac{1}{1+x^4}dx$

Sol. $\int \frac{1}{1+x^4}dx$

$$= \frac{1}{2}\int \frac{2}{1+x^4}dx = \frac{1}{2}\int \frac{1+x^2+1-x^2}{1+x^4}dx$$

$$=\frac{1}{2}\int\left(\frac{1+x^2}{1+x^4}+\frac{1-x^2}{1+x^4}\right)dx$$

$$=\frac{1}{2}\int\left(\frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}+\frac{\frac{1}{x^2}-1}{x^2+\frac{1}{x^2}}\right)dx$$

$$=\frac{1}{2}\left(\int\frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+\left(\sqrt{2}\right)^2}dx-\int\frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2-\left(\sqrt{2}\right)^2}dx\right)$$

$$=\frac{1}{2}\left(\frac{1}{\sqrt{2}}\tan^{-1}\frac{x-\frac{1}{x}}{\sqrt{2}}-\frac{1}{2\sqrt{2}}\log\frac{x+\frac{1}{x}-\sqrt{2}}{x+\frac{1}{x}+\sqrt{2}}\right)+c$$

$$=\frac{1}{2\sqrt{2}}\left(\tan^{-1}\frac{x^2-1}{x\sqrt{2}}-\frac{1}{2}\log\frac{x^2+1-\sqrt{2}}{x^2+1+\sqrt{2}}\right)+c$$

23. Find the area bounded between the curves $y^2=4ax,\ x^2=4by\left(a>0,b>0\right)$**.**

Sol: Equations of the given curves are

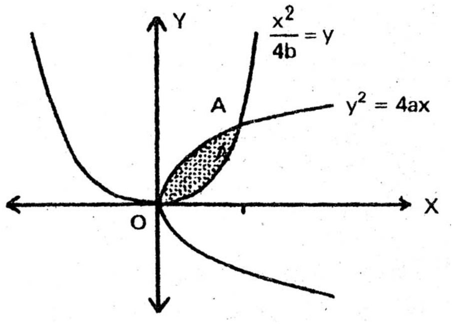
$y^2=4ax$ ....(1)

$x^2=4by$ ....(2)

From equation (2) y = $\frac{x^2}{4b}$

Substituting in (1) $\left(\frac{x^2}{4b}\right)^2=4ax$

$x^4=\left(16b^2\right)|\,4ax\,|$



$$x\left[x^3 - 64b^2a\right] = 0$$

X = 0, x = 4 $\left(b^2a\right)^{1/3}$

Area bounded will be

$$= \int_0^{4\left(b^2a\right)^{1/3}} \left[\sqrt{4ax} - \frac{x^2}{4b}\right] dx$$

$$= {}_0^{4\left(b^2a\right)^{1/3}} \left[(4a)^{1/2} x^{3/2} . \frac{2}{3} - \frac{x^3}{12b}\right]$$

$$= \left[(4a)^{1/2} 8\left(b^2a\right)^{\frac{1}{3}.\frac{3}{2}} \frac{2}{3} - \frac{4^3\left(b^2a\right)^{3.\frac{1}{3}}}{12b}\right]$$

$$= \left[2ab\frac{16}{3} - \frac{64.b^2a}{12b}\right] = ab\left(\frac{32}{3} - \frac{16}{3}\right)$$

$$= \frac{16}{3} \text{ ab sq.units}$$

24. $(y^2 - 2xy)dx + (2xy - x^2)dy = 0$

Sol. $(y^2 - 2xy)dx + (2xy - x^2)dy = 0$

$(2xy - x^2)dy = -(y^2 - 2xy)dx$

$$\frac{dy}{dx}=\frac{2xy-y^2}{2xy-x^2}$$

Put $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$

$$v+x\frac{dv}{dx}=\frac{2vx^2-v^2x^2}{2vx^2-x^2}=\frac{x^2(2v-v^2)}{x^2(2v-1)}$$

$$x\frac{dv}{dx}=\frac{2v-v^2}{2v-1}-v$$

$$=\frac{2v-v^2-2v^2+v}{2v-1}=\frac{3v(1-v)}{2v-1}$$

$$\int\frac{2v-1}{v(1-v)}dv=3\int\frac{dx}{x} \qquad \ldots (1)$$

Let $\frac{2v-1}{v(1-v)}=\frac{A}{v}+\frac{B}{1-v}$

$2v-1=A(1-v)+Bv$

$v=0\Rightarrow -1=A\Rightarrow A=-1$

$v=1\Rightarrow 1=B\Rightarrow B=1$

$$\int\left(-\frac{1}{v}+\frac{1}{1+v}\right)dv=3\int\frac{dx}{x}$$

$-\log v-\log(1-v)=3\log x+\log c$

$\log\frac{1}{v(1-v)}=\log cx^3$

$$\frac{1}{v(1-v)}=cx^3\Rightarrow v(1-v)=\frac{1}{cx^3}$$

$$\frac{y}{x}\left(1-\frac{y}{x}\right)=\frac{1}{cx^3}\Rightarrow\frac{y}{x}\left(\frac{x-y}{x}\right)=\frac{1}{cx^3}$$

$$xy(x-y)=\frac{1}{c}=k\Rightarrow xy(y-x)=-\frac{1}{c}=k$$