MATHEMATICS PAPER IIA

TIME: 3hrs **Max. Marks.**75

Note: This question paper consists of three sections A, B and C.

SECTION A

Very Short Answer Type Questions. 10 X 2 =20

1. Find the maximum or minimum of the expression $x^2 - x + 7$as x varies over R.

   $x^2 - x + 7$

2. Given that the roots of $x^3 + 3px^2 + 3px + r = 0$ are in A.P., then show that

$2p^2 - 3qp + r = 0$

3. Find the least positive integer n, satisfying $\left(\frac{1+i}{1-i}\right)^n = 1$.

4. Show that the complex numbers z satisfying $z^2 + \bar{z}^2 = 2$ constitute a hyperbola.

5. Find all values of $\left(1 - i\sqrt{3}\right)^{\frac{1}{3}}$

6. How many four digited numbers can be formed using the digits 1, 2, 5, 7, 8, 9? How many of them begin with 9 and end with 2?

7. Find the number of diagonals of a polygon with 12 sides.

8. Find the term independent of x (that is the constant term) in the expansion of $\left(\frac{\sqrt{x}}{3} + \frac{3}{2x^2}\right)^{10}$.

9. A probability distribution function of a discrete random variable is zero except at the points x = 0, 1, 2. At these points it has the value $p(0) = 3c^3$, $p(1) = 4c - 10c^2$, $p(2) = 5c - 1$ for some $c > 0$. Find the value of c.

10. Find the mean for the following distribution.

x_i	10	11	12	13
f_i	3	12	18	12

SECTION -B

Short Answer Type Questions.

Answer Any Five of the Following 5 X 4 = 20

11. If the roots of $ax^2 + bx + c = 0$ are real and equal to $\alpha = \frac{-b}{2a}$, then $\alpha \neq x \in R$, $ax^2 + bx + c$ and a will have same sign.

12. If the point P denotes the complex number z = x + iy in the Argand plane and if $\frac{z-i}{z-1}$ is a purely imaginary number, find the locus of P.

13. Resolve $\frac{x^3}{(2x-1)(x+2)(x-3)}$ into partial fractions.

14. Find the number of 4-digit numbers which can be formed using the digits 0, 2, 5, 7, 8 that are divisible by (i) 2 (ii) 4 when repetition is allowed.

15. Find the number of ways of selecting 11 member cricket team from 7 batsmen, 5 bowlers and 3 wicket keepers with at least 3 bowlers and 2 wicket keepers.

16. The probabilities of three events A, B, C are such that P(A) = 0.3, P(B) = 0.4, P(C) = 0.8, $P(A \cap B) = 0.08$, $P(A \cap C) = 0.28$, $P(A \cap B \cap C) = 0.09$ and $P(A \cup B \cup C) \geq 0.75$. Show that $P(B \cap C)$ lies in the interval [0.23, 0.48].

17. If A, B, C are three independent events such that $P(A \cap B^C \cap C^C) = \frac{1}{4}$

$P(A^C \cap B \cap C^C) = \frac{1}{8}, P(A^C \cap B^C \cap C^C) = \frac{1}{4}$ then find P(A), P(B) and P(C).

SECTION- C

Long Answer Type Questions.

Answer Any Five of the Following 5 X 7 =35

18. Solve the equation $6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$

19. If n is an integer then show that $(1+\cos\theta + i\sin\theta)^n + (1+\cos\theta - i\sin\theta)^n =$

$2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$.

20. If n is a positive integer and x is any non-zero real number, then prove that

$$C_0 + C_1\frac{x}{2} + C_2 \cdot \frac{x^2}{3} + C_3 \cdot \frac{x^3}{4} + \ldots + C_n \cdot \frac{x^n}{n+1} = \frac{(1+x)^{n+1} - 1}{(n+1)x}$$

$$C_0 + C_1\frac{x}{2} + C_2 \cdot \frac{x^2}{3} + C_3 \cdot \frac{x^3}{4} + \ldots + C_n \cdot \frac{x^n}{n+1} = \frac{(1+x)^{n+1} - 1}{(n+1)x}$$

21. If the coefficients of x^9, x^{10}, x^{11} in the expansion of $(1+x)^n$ are in A.P. then prove that $n^2 - 41n + 398 = 0$.

22. In a shooting test the probability of A, B, C hitting the targets are $\frac{1}{2}, \frac{2}{3}$ and $\frac{3}{4}$ respectively. It all of them fire at the same target. Find the probability that

i) Only one of them hits the target. ii) At least one of them hits the target.

23. The range of a random variable x is {0, 1, 2}.

Given that $p(x = 0) = 3c^3$, $p(x = 1) = 4c - 10c^2$, $p(x = 2) = 5c - 1$

i) Find the value of c

ii) $p(x < 1)$, $p(1 < x \leq 3)$

24. Find the mean and variance using the step deviation method of the following tabular data, giving the age distribution of 542 members.

Age in years (x_i)	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of members (f_i)	3	61	132	153	140	51	2

Maths IIA Paper 2 - Solutions

1. Find the maximum or minimum of the following expression as x varies over R.

i) $x^2 - x + 7$

iii) $2x + 5 - 3x^2$

iv) $ax^2 + bx + a$ (a, b $\in$ R and a $\neq$ 0)

Sol: i) $y = x^2 - x + 7$

$$y = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} + 7$$

$$y = \left(x - \frac{1}{2}\right)^2 + \frac{27}{4}$$

$$y_{min} = \frac{27}{4}$$

2. Given that the roots of $x^3 + 3px^2 + 3px + r = 0$ are in A.P.,then show that

$2p^2 - 3qp + r = 0$

Sol: Given equation is $x^3 + 3px^2 + 3px + r = 0$

The roots are in A.P.

Suppose $a - d, a, a + d = -3p$

$3a = -3p \Rightarrow a = -p$ (1)

$\because 'a'\ x^3 + 3px^2 + 3qx + r = 0$

$\Rightarrow a^3 + 3pa^2 + 3qa + r = 0$

But $a = -p$

$\Rightarrow -p^3 + 3p(-p)^2 + 3q(-p) + r = 0$

$\Rightarrow 2p^3 - 3pq + r = 0$ is the required condition

3. Find the least positive integer n, satisfying $\left(\frac{1+i}{1-i}\right)^n = 1$.

Sol: i) $\left(\frac{1+i}{1-i}\right)^n = 1$

$$\left(\frac{(1+i)(1+i)}{(1-i)(1+i)}\right)^n = 1$$

$$\left(\frac{2i}{2}\right)^n = 1$$

$$i^n = 1$$

$$(\because i^n = 1 = -1 \times -1 = i^2 \times i^2 = i^4)$$

$$n = 4$$

$$i^4 - 1$$

Least value of n = 4.

4. Show that the complex numbers z satisfying $z^2 + \overline{z}^2 = 2$ constitute a hyperbola.

Sol. Substituting z = x + iy in the given equation $z^2 + \overline{z}^2 = 2$, we obtain the Cartesian form of the given equation.

$$\therefore (x+iy)^2 + (x-iy)^2 = 2$$

i.e., $x^2 - y^2 + 2ixy + x^2 - y^2 - 2ixy = 2$

or $2x^2 - 2y^2 = 2$ i.e., $x^2 - y^2 = 1$.

Since this equation denotes a hyperbola all the complex numbers satisfying $z^2 + \overline{z}^2 = 2$ constitute the hyperbola $x^2 - y^2 = 1$.

5. Find all values of $\left(1 - i\sqrt{3}\right)^{\frac{1}{3}}$

$$\left(1 - i\sqrt{3}\right)^{\frac{1}{3}} = \left\{2\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)\right\}^{\frac{1}{3}}$$

$$= 2^{\frac{1}{3}}\left\{\cos\left(\frac{-\pi}{3}\right) + i\sin\left(\frac{-\pi}{3}\right)\right\}^{\frac{1}{3}}$$

$$= 2^{\frac{1}{3}}\left\{\cos\frac{\left(2k\pi-\frac{\pi}{3}\right)}{3} + isn\left(\frac{2k\pi-\frac{\pi}{3}}{3}\right)\right\}\ k = 0, 1, 2$$

$$= 3\sqrt{2}\ cis(6k-1)\frac{\pi}{9}\ \ k = 0, 1, 2$$

6. How many four digited numbers can be formed using the digits 1, 2, 5, 7, 8, 9? How many of them begin with 9 and end with 2?

Sol: The number of four digited numbers that can be formed using the given digits 1, 2, 5, 7, 8, 9 is ${}^6P_4 = 360$. Now, the first place and last place can be filled with 9 and 2 in one way.

9			2

The remaining 2 places can be filled by the remaining 4 digits 1, 5, 7, 8. Therefore these two places can be filled in ${}^4P_2$ ways. Hence, the required number of ways =

$1.{}^4P_2 = 12.$

7. Find the number of diagonals of a polygon with 12 sides.

Sol: Number of sides of a polygon = 12

Number of diagonals of a n-sided polygon

$= {}^nC_2 - n$

$\therefore$ Number of diagonals of 12 sided polygon

$= {}^{12}C_2 - 12 = 54$.

8. Find the term independent of x (that is the constant term) in the expansion of $\left(\frac{\sqrt{x}}{3} + \frac{3}{2x^2}\right)^{10}$.

Sol. $T_{r+1} = {}^{10}C_r\left(\sqrt{\frac{x}{3}}\right)^{10-r}\left(\frac{3}{2x^2}\right)^r \qquad = \frac{{}^{10}C_r \cdot 3^{\frac{3r-10}{2}}}{2^r} \cdot x^{\frac{10-5r}{2}}$

To find the term independent of x, put

$$\frac{10-5r}{2}=10 \Rightarrow r=2$$

$$\therefore T_3=\frac{{}^{10}C_2 3^{\frac{6-10}{2}}}{2^2}\cdot x^{\frac{10-10}{2}}=\frac{{}^{10}C_2 3^{-2} x^0}{2^2}=\frac{5}{4}$$

9. A probability distribution function of a discrete random variable is zero except at the points x = 0, 1, 2. At these points it has the value p (0) = $3c^3$, p(1) = 4c – $10c^2$, p(2) = 5c – 1 for some c > 0. Find the value of c.

Sol.

P(x = 0) + p(x = 1) + p(x = 2) = 1

$3c^3 + 4c - 10c^2 + 5c - 1 = 1$

$3c^3 - 10^2 + 9c - 2 = 0$-----------(1)

Put c = 1, then 3 – 10+9 –2 = 12 – 12 = 0

C = 1 satisfy the above equation

C = 1 ⇒ p(x = 1) = 4-10= -6 which is not possible.

Dividing (1) with c – 1,

We get $3c^2 - 7c + 2 = 0$

⇒ (c – 2) (3c – 1) = 0

c = 2 or c = 1/3

c = 2 ⇒ p(x = 0) = 3.2^3 = 24 which is not possible

∴ c = 1/3

10. Find the mean for the following distribution.

i)

x_i	10	11	12	13
f_i	3	12	18	12

Sol. i)

x_i	f_i	$f_i x_i$
10	3	30
11	12	132
12	18	216
13	12	156
	N = 45	$\Sigma f_i x_i = 534$

$\therefore$ Mean $(\bar{x}) = \frac{\Sigma f_i x_i}{N} = \frac{534}{45} = 11.87$

11. If the roots of $ax^2 + bx + c = 0$are real and equal to $\alpha = \frac{-b}{2a}$, then $\alpha \neq x \in R$, $ax^2 + bx + c$ and a will have same sign.

Proof :

The roots of $ax^2 + bx + c = 0$ are real and equal

$\Rightarrow b^2 = 4ac \Rightarrow 4ac - b^2 = 0$

$$\frac{ax^2 + bx + c}{a} = x^2 + \frac{b}{a}x + \frac{c}{a}$$

$$= \left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \frac{b^2}{4a^2}$$

$$=\left(x+\frac{b}{2a}\right)^2+\frac{4ac-b^2}{4a^2}$$

$$=\left(x+\frac{b}{2a}\right)^2 > 0 \text{ for } x \neq \frac{-b}{2a}=\alpha$$

for $\alpha \neq x \in R, ax^2+bx+c$ and a have the same sign

12. If the point P denotes the complex number z = x + iy in the Argand plane and if $\frac{z-i}{z-1}$ is a purely imaginary number, find the locus of P.

Sol: We note that $\frac{z-i}{z-1}$ is not defined if z = 1.

Since, z = x + iy,

$$\frac{z-i}{z-1}=\frac{x+iy-i}{x+iy-1}=\frac{x+i(y-1)}{x-1+iy}$$

$$=\frac{[x+i(y-1)][(x-1)-iy]}{[(x-1)+iy]\,[(x-1)-iy]}$$

$$=\frac{x^2+y^2-x-y}{(x-1)^2+y^2}+i\left(\frac{1-x-y}{(x-1)^2+y^2}\right)$$

$\frac{z-i}{z-1}$ will be purely imaginary, if and $z \neq 1$ and $\frac{x^2+y^2-x-y}{(x-1)^2+y^2}=0$.

i.e., $x^2+y^2-x-y=0$ and $(x,y)\neq(1,0)$.

$\therefore$ The locus of P is the circle $x^2+y^2-x-y=0$ excluding the point (1, 0).

13. Resolve $\frac{x^3}{(2x-1)(x+2)(x-3)}$ into partial fractions.

Sol. $\frac{x^3}{(2x-1)(x+2)(x-3)}=$

$$\frac{1}{2}+\frac{A}{2x-1}+\frac{B}{x+2}+\frac{C}{x-3}$$

Multiplying with 2(2x – 1)(x + 2)(x – 3)

$$2x^3 = (2x-1)(x+2)(x-3)+2A(x+2)$$
$$(x-3)+2B(2x-1)(x-3)+2C(2x-1)(x+2)$$

Put $x=\frac{1}{2}, 2\left(\frac{1}{8}\right)=2A\left(\frac{5}{2}\right)\cdot\left(-\frac{5}{2}\right)\Rightarrow A=-\frac{1}{50}$

Put x = –2, 2(–8) = 2B(–5)(–5) $\Rightarrow$ B = $\frac{-8}{25}$

Put x = 3, 2(27) = 2C(5)(5) $\Rightarrow$ C = $\frac{27}{25}$

$$\therefore \frac{x^3}{(2x-1)(x+2)(x-3)}=\frac{1}{2}-\frac{1}{50(2x-1)}-\frac{8}{25(x+2)}+\frac{27}{25(x-3)}$$

14. Find the number of 4-digit numbers which can be formed using the digits 0, 2, 5, 7, 8 that are divisible by (i) 2 (ii) 4 when repetition is allowed.

Sol: Given digits are 0, 2, 5, 7, 8.

i) Divisible by 2:

The thousand's place of 4 digit number when repetition is allowed can be filled in 4 ways. (Using non-zero digits)

The 4-diti number is divisible by 2, when the units place is an even digit. This can be done in 3 ways.

The remaining 2 places can be filled by 5 ways each i.e., 5^2 = 25 ways.

$\therefore$ Number of 4 digit numbers which are divisible by 2 is 4 × 3 × 25 = 300.

ii) Divisible by 4:

A number is divisible by 4 only when the number in last two places (ten's and unit's) is a multiple of 4.

As repletion is allowed the last two places should be filled with one of the following :

00, 08, 20, 28, 52, 72, 80, 88

This can be done is 8 ways.

Thousand's place is filled in 4 ways.

(i.e., using non-zero digits)

Hundred's placed can be filled in 5 ways.

$\therefore$ Total number of 4 digit numbers formed

$= 8 \times 4 \times 5 = 160.$

15. Find the number of ways of selecting 11 member cricket team from 7 batsmen, 5 bowlers and 3 wicket keepers with atleast 3 bowlers and 2 wicket keepers.

Sol.

Bowlers (5)-	Wickt Keepers (3)	Bats men (7)	Number of ways of selecting them
3	2	6	${}^5C_3 \times {}^3C_2 \times {}^7C_6 = 210$
4	2	5	${}^5C_4 \times {}^3C_2 \times {}^7C_5 = 315$
5	2	4	${}^5C_5 \times {}^3C_2 \times {}^7C_4 = 105$
3	3	5	${}^5C_3 \times {}^3C_3 \times {}^7C_5 = 210$
4	3	4	${}^5C_4 \times {}^3C_3 \times {}^7C_4 = 175$
5	3	3	${}^5C_5 \times {}^3C_3 \times {}^7C_3 = 35$

The required teams can contain the following compositions

Therefore, the number of required ways

$= 210 + 315 + 105 + 210 + 175 + 35 \quad = 1050.$

16. The probabilities of three events A, B, C are such that P(A) = 0.3, P(B) = 0.4, P(C) = 0.8, P(A ∩ B) = 0.08, P(A ∩C) = 0.28, P(A ∩ B ∩ C) = 0.09 and P(A ∪ B ∪ C) ≥ 0.75. Show that P(B ∩ C) lies in the interval [0.23, 0.48].

Sol. $P(A \cup B \cup C) \geq 0.75$

$0.75 \leq P(A \cup B \cup C) \leq 1$

$\Rightarrow 0.75 \leq P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C) \leq 1$

$\Rightarrow 0.75 \leq 0.3+0.4+0.8-0.08-0.28-P(B\cap C)+0.09 \leq 1$

$\Rightarrow 0.75 \leq 1.23-P(B\cap C) \leq 1$

$\Rightarrow -0.75 \geq P(B\cap C)-1.23 \geq -1$

$\Rightarrow 0.48 \geq P(B\cap C) \geq 0.23$

$\Rightarrow 0.23 \leq P(B\cap C) \leq 0.48$

$\therefore$ P(B ∩ C) lies in the interval [0.23, 0.48].

17. If A, B, C are three independent events such that $P(A\cap B^C \cap C^C)=\frac{1}{4}$

$P(A^C \cap B\cap C^C)=\frac{1}{8}, P(A^C \cap B^C \cap C^C)=\frac{1}{4}$ **then find P(A), P(B) and P(C).**

Sol. Since A, B, C are independent events.

$P(A\cap B^C \cap C^C)=\frac{1}{4}$

$\Rightarrow P(A)\cdot P(B^C)\cdot P(C^C)=\frac{1}{4}$ ...(1)

$P(A^C \cap B\cap C^C)=\frac{1}{8}$

$\Rightarrow P(A^C)\cdot P(B)\cdot P(C^C)=\frac{1}{8}$ ...(2)

$$P(A^C \cap B^C \cap C^C) = \frac{1}{4}$$

$$\Rightarrow P(A^C) \cdot P(B^C) \cdot P(C^C) = \frac{1}{4} \quad ...(3)$$

$$\frac{(1)}{(3)} \Rightarrow \frac{P(A)}{P(A^C)} = \frac{1/4}{1/4} = 1$$

$$\Rightarrow \frac{P(A)}{1-P(A)} = 1 \Rightarrow P(A) = 1 - P(A)$$

$$\Rightarrow 2P(A) = 1 \Rightarrow P(A) = \frac{1}{2}$$

$$\frac{(2)}{(3)} \Rightarrow \frac{P(B)}{P(B^C)} = \frac{1/8}{1/4} \Rightarrow \frac{P(B)}{1-P(B)} = \frac{1}{2}$$

$$\Rightarrow 2P(B) = 1 - P(B) \Rightarrow 3P(B) = 1$$

$$\therefore P(B) = \frac{1}{3}$$

From (1) $\Rightarrow P(A) \cdot P(B^C) \cdot P(C^C) = \frac{1}{4}$

$$\Rightarrow \left(\frac{1}{2}\right)\left(1 - \frac{1}{3}\right) P(C^C) = \frac{1}{4}$$

$$\Rightarrow P(C^C) = \frac{1}{4} \times 2 \times \frac{3}{2} = \frac{3}{4}$$

$$\therefore P(C) = 1 - P(C^C) = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\therefore P(A) = \frac{1}{2},\ P(B) = \frac{1}{3},\ P(C) = \frac{1}{4}$$

18. Solve the equation $6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$

Sol: Given equation is $6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$ is a reciprocal equation of second class and of even degree

$\therefore x^2 - 1$ is a factor of

$$6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$$

1	6	−25	31	0	−31	25	−6	
	–	6	−19	12	12	−19	6	
−1	6	−19	12	12	−19	6	0	
	–	−6	25	−37	25	−6		
	6	−25	37	−25	6	0		

$$6x^4 - 25x^3 + 37x^2 - 25x + 6 = 0$$

$$\Rightarrow 6x^2 - 25x + 37 - \frac{25}{x} + \frac{6}{x^2} = 0$$

$$\Rightarrow 6\left(x^2 + \frac{1}{x^2}\right) - 25\left(x + \frac{1}{x}\right) + 37 = 0 \text{-----------}(1)$$

Put $x + \frac{1}{x} = y \Rightarrow x^2 + \frac{1}{x^2} = y^2 - 2$

$\therefore$ (1) Becomes $6\left(y^2 - 2\right) - 25\left(y\right) + 37 = 0$

$$\Rightarrow 6y^2 - 12 - 25y + 37 = 0$$

$$\Rightarrow 6y^2 - 25 + 25 = 0$$

$$\Rightarrow 6y^2 - 15y - 10y + 25 = 0$$

$$\Rightarrow 3y(2 - 5) - 5(2y - 5) = 0$$

$$\Rightarrow (2y - 5)(3y - 5) = 0$$

$$\Rightarrow y = \frac{5}{2}, \frac{5}{3}$$

Case (i) : $y = \frac{5}{2} \Rightarrow x + \frac{1}{x} = \frac{5}{2} \Rightarrow \frac{x^2+1}{x} = \frac{5}{2}$

$\Rightarrow 2x^2 + 2 = 5x$

$\Rightarrow 2x^2 - 5x + 2 = 0$

$\Rightarrow (2x-1)(x-2) = 0$

$\therefore x = \frac{1}{2}, 2$

Case (ii) $y = \frac{5}{2} \Rightarrow x + \frac{1}{x} = \frac{5}{3} \Rightarrow \frac{x^2+1}{x} = \frac{5}{3}$

$\Rightarrow 3x^2 + 3 = 5x$

$\Rightarrow 3x^2 - 5x + 3 = 0$

$\Rightarrow x = \frac{5 \pm \sqrt{25-36}}{6}$

$\therefore x = \frac{5 \pm \sqrt{11}i}{6}$

$\therefore$ The roots of the given equation are

$\pm 1, \frac{1}{2}, 2, \frac{5 \pm \sqrt{11}i}{6}$

19. If n is an integer then show that $(1+\cos\theta + i\sin\theta)^n + (1+\cos\theta - i\sin\theta)^n =$ $2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$.

Sol. L.H.S. =

$(1+\cos\theta + i\sin\theta)^n + (1+\cos\theta - i\sin\theta)^n =$

$$= \left(2\cos^2\frac{\theta}{2} + 2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)^n + \left(2\cos^2\frac{\theta}{2} - 2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)^n$$

$$= 2^n\cos^n\frac{\theta}{2}\left[\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)^n + \left(\cos\frac{\theta}{2} - i\sin\frac{\theta}{2}\right)^n\right]$$

$$= 2^n\cos^n\frac{\theta}{2}\left(\cos\frac{n\theta}{2} + i\sin\frac{n\theta}{2} + \cos\frac{n\theta}{2} - i\sin\frac{n\theta}{2}\right)$$

$$= 2^n\cos^n\frac{\theta}{2}\left(2\cos\frac{n\theta}{2}\right)$$

$$= 2^{n+1}\cos^n\frac{\theta}{2}\cos\frac{n\theta}{2} = \text{R.H.S.}$$

20. If n is a positive integer and x is any non-zero real number, then prove that

$$C_0 + C_1\frac{x}{2} + C_2\cdot\frac{x^2}{3} + C_3\cdot\frac{x^3}{4} + \ldots + C_n\cdot\frac{x^n}{n+1} = \frac{(1+x)^{n+1} - 1}{(n+1)x}$$

Sol. $C_0 + C_1\frac{x}{2} + C_2\cdot\frac{x^2}{3} + C_3\cdot\frac{x^3}{4} + \ldots + C_n\cdot\frac{x^n}{n+1}$

$$= {}^nC_0 + \frac{1}{2}\,{}^nC_1 x + \frac{1}{3}\,{}^nC_2 x^2 + \ldots + \frac{1}{n+1}\,{}^nC_n x^n$$

$$= 1 + \frac{n}{1!}\frac{x}{2} + \frac{n(n-1)}{2!}\frac{x^2}{3} + \ldots\ldots$$

$$= 1 + \frac{n}{2!}x^1 + \frac{n(n-1)}{3!}x^2 + \ldots\ldots$$

$$= \frac{1}{(n+1)x}\left[\frac{(n+1)x^1}{1!} + \frac{(n+1)n}{2!}x^2 + \frac{(n+1)n(n-1)}{3!}x^3 + \ldots\right]$$

$$= \frac{1}{(n+1)x}\left[{}^{(n+1)}C_1 x + {}^{(n+1)}C_2 x^2 + {}^{(n+1)}C_3 x^3 + \ldots\right]$$

$$= \frac{1}{(n+1)x}\left[1 + {}^{n+1}C_1 x + {}^{n+1}C_2 x^2 + \ldots + {}^{n+1}C_{n+1} x^{n+1} - 1\right]$$

$$= \frac{1}{(n+1)x}\left[(1+x)^{n+1} - 1\right]$$

21. $C_0 + C_1 \frac{x}{2} + C_2 \cdot \frac{x^2}{3} + C_3 \cdot \frac{x^3}{4} + \ldots + C_n \cdot \frac{x^n}{n+1} = \frac{(1+x)^{n+1} - 1}{(n+1)x}$

If the coefficients of x^9, x^{10}, x^{11} in the expansion of $(1+x)^n$ are in A.P. then prove that $n^2 - 41n + 398 = 0$**.**

Sol: Coefficient of x^r in the expansion $(1 - x)^n$ is ${}^nC_r$.

Given coefficients of x^9, x^{10}, x^{11} in the expansion of $(1 - x)^n$ are in A.P., then

$$2({}^nC_{10}) = {}^nC_9 + {}^nC_{11}$$

$$\Rightarrow 2\frac{n!}{(n-10)!10!} = \frac{n!}{(n-9)!9!} + \frac{n!}{(n-11)!+11!}$$

$$\Rightarrow \frac{2}{10(n-10)} = \frac{1}{(n-9)(n-10)} + \frac{1}{11\times 10}$$

$$\Rightarrow \frac{2}{(n-10)10} = \frac{110 + (n-9)(n-10)}{110(n-9)(n-10)}$$

$$\Rightarrow 22(n-9) = 110 + n^2 - 19n + 90$$

$$\Rightarrow n^2 - 41n + 398 = 0$$

22. In a shooting test the probability of A, B, C hitting the targets are $\frac{1}{2}, \frac{2}{3}$ and $\frac{3}{4}$ espectively. It all of them fire at the same target. Find the probability that

i) Only one of them hits the target.

ii) At least one of them hits the target.

Sol. The probabilities that A, B, C hitting the targets are denoted by

$$P(A) = \frac{1}{2},\ P(B) = \frac{2}{3} \text{ and } P(C) = \frac{3}{4}$$

$$\therefore P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$$

$$P(\bar{B}) = 1 - P(B) = 1 - \frac{2}{3} = \frac{1}{3}$$

$$P(\bar{C}) = 1 - P(C) = 1 - \frac{3}{4} = \frac{1}{4}$$

i) Probability that only one of them hits the target

$$= P(A \cap \bar{B} \cap \bar{C}) + P(\bar{A} \cap B \cap \bar{C}) + P(\bar{A}\bar{B}C)$$

$$= P(A) \cdot P(\bar{B}) \cdot P(\bar{C}) + P(\bar{A}) \cdot P(B) \cdot P(\bar{C}) + P(\bar{A}) \cdot P(\bar{B}) \cdot P(C)$$

($\because$ A, B, C are independent events)

$$= \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{3}{4}$$

$$= \frac{1+2+3}{24} = \frac{6}{24} = \frac{1}{4}$$

ii) Probability that atleast one of them hits the target = $P(A \cup B \cup C)$

= 1 – Probability that none of them hits the target.

$= 1 - P(\bar{A}\ \bar{B}\ \bar{C})$

$= 1 - P(\bar{A}) \cdot P(\bar{B}) \cdot P(\bar{C})$

$= 1 - \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} = 1 - \frac{1}{24} = \frac{23}{24}$

23. The range of a random variable x is {0, 1, 2}. Given that p(x = 0) = $3c^3$, p(x = 1) = 4c – $10c^2$, p(x = 2) = 5c – 1

i) Find the value of c

ii) p(x < 1), p(1 < x ≤ 3)

Sol. P(x = 0) + p(x = 1) + p(x = 2) = 1

$3c^3 + 4c - 10c^2 + 5c - 1 = 1$

$3c^3 - 10c^2 + 9c - 2 = 0$

C = 1 satisfy this equation

C = 1 ⇒ p(x = 0) = 3 which is not possible dividing with c – 1, we get

$3c^2 - 7c + 2 = 0 \Rightarrow (c - 2)(3c - 1) = 0$

d = 2 or c = 1/3

c = 2 ⇒ p(x = 0) = 3.2^3 = 24 which is not possible

∴ c = 1/3

i) p(x < 1) = p(x =0)

$= 3.c^3 = \left(\frac{1}{3}\right)^3 = 3.\frac{1}{27} = \frac{1}{9}$

ii) p(1 < x ≤ 2) = p(x = 2) = 5c – 1

$= \frac{5}{3} - 1 = \frac{2}{3}$

iii) $p(0 < x \leq 3) = p(x = 1) + p(x = 2)$

$= 4c - 10c^2 + 5c - 1$

$= 9c - 10c^2 - 1 = 9.\frac{1}{3} - 10.\frac{1}{9} - 1$

$= 3 - \frac{10}{9} - 1 = 2 - \frac{10}{9} = \frac{8}{9}$

24. Find the mean and variance using the step deviation method of the following tabular data, giving the age distribution of 542 members.

Age in years (x_i)	**20-30**	**30-40**	**40-50**	**50-60**	**60-70**	**70-80**	**80-90**
No. of members (f_i)	**3**	**61**	**132**	**153**	**140**	**51**	**2**

Sol.

Age in years C.I.	Mid point (x_i)	(f_i)	$d_i = \frac{x_i - A}{C}$ A = 55, C = 10	$f_i d_i$	d_i^2	$f_i d_i^2$
20-30	25	3	–3	–9	9	27
30-40	35	61	–2	–122	4	244
40-50	45	132	–1	–132	1	132
50-60	55→A	153	0	0	0	0
60-70	65	140	1	140	1	140
70-80	75	51	2	102	4	204
80-90	85	2	3	6	9	18
	N=542			–15	28	765

$$\text{Mean } (\overline{x}) = A + \frac{\Sigma f_i d_i}{N} \times C = 55 + \frac{-15}{542} \times 10 = 55 - 0.277 = 54.723$$

$$\text{Variance } (\mu) = \frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2 = \frac{765}{542} - \left(\frac{-15}{542}\right)^2 = \frac{765}{542} - \frac{225}{(542)^2}$$

$$= \frac{542 \times 765 - 225}{(542)^2} = \frac{414630 - 225}{293764} = \frac{414405}{293764} = 1.4106$$

$$V(\mu) = V\left(\frac{X - A}{C}\right) = \left(\frac{1}{C}\right)^2 \cdot V(X) \qquad \left[\because V(ax + n) = a^2 \cdot V(x)\right]$$

$$V(X) = C^2 \cdot V(\mu) = 100 \times 1.4106 = 141.06.$$