

DEMOVIRE'S THEOREM AND
TRIGONOMETRIC EXPANSION
PREVIOUS EAMCET BITS

1. A value of n such that $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^n = 1$ is [EAMCET 2007]

1) 12 2) 3 3) 2 4) 1

Ans: 1

Sol. $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^n = 1 \Rightarrow \text{cis } \frac{n\pi}{6} = 1$

$\therefore n = 12$

2. If α is a non-real root of $x^6 = 1$, then $\frac{\alpha^5 + \alpha^3 + \alpha + 1}{\alpha^2 + 1} =$ [EAMCET 2005]

1) α^2 2) 0 3) $-\alpha^2$ 4) α

Ans: 3

Sol. $x^6 = 1 \Rightarrow \text{let } \alpha = \omega$

$$\frac{\alpha^5 + \alpha^3 + \alpha + 1}{\alpha^2 + 1} = \frac{\omega^5 + \omega^3 + \omega + 1}{\omega^2 + 1}$$

$$= -\omega^2 = -\alpha^2$$

3. If $x_n = \cos \frac{\pi}{2^n} + i \sin \frac{\pi}{2^n}$, then $\prod_{n=1}^{\infty} x_n =$ [EAMCET 2004]

1) -1 2) 1 3) $\frac{1}{\sqrt{2}}$ 4) $\frac{i}{\sqrt{2}}$

Ans: 1

Sol. $x_n = \text{cis } \frac{\pi}{2^n} \Rightarrow x_1 x_2 x_3 \dots$

$$= \text{cis } \frac{(\pi/2)}{1 - \frac{1}{2}} = \text{cis } \pi = -1$$

4. If $\sin 6\theta = 32 \cos^2 \theta \sin \theta - 32 \cos^3 \theta \sin \theta + 3x$, then $x = \dots\dots\dots$ [EAMCET 2003]

1) $\cos \theta$ 2) $\cos 2\theta$ 3) $\sin \theta$ 4) $\sin 2\theta$

Ans: 4

Sol. $\sin 6\theta = 2 \sin 3\theta \cos 3\theta$
 $= 2(3 \sin \theta - 4 \sin^3 \theta)(4 \cos^3 \theta - 3 \cos \theta)$
 $= 32 \cos^5 \theta \sin \theta - 32 \cos^3 \theta \sin \theta + 3 \sin 2\theta$
 $\therefore x = \sin 2\theta$

5. If $x_n = \cos\left(\frac{\pi}{4^n}\right) + i \sin\left(\frac{\pi}{4^n}\right)$, $x_1 x_2 x_3 \dots \dots \dots \infty =$ [EAMCET 2002]

1) $\frac{1+i\sqrt{3}}{2}$

2) $\frac{-1+i\sqrt{3}}{2}$

3) $\frac{1-i\sqrt{3}}{2}$

4) $\frac{-1-i\sqrt{3}}{2}$

Ans: 1

Sol. $x_1 x_2 \dots \dots \dots \infty$

$$= \text{cis } \frac{\pi}{4} \cdot \text{cis } \frac{\pi}{4^2} \cdot \text{cis } \frac{\pi}{4^3} \dots \dots \infty$$

$$= \text{cis} \left(\frac{\pi}{4} + \frac{\pi}{4^2} + \dots \dots \infty \right) = \text{cis} \left(\frac{\pi/4}{1-1/4} \right)$$

$$= \text{cis } \frac{\pi}{3} = \frac{1+i\sqrt{3}}{2}$$

6 If $\theta = \pi/6$, then the tenth term of $1 + (\cos \theta + i \sin \theta) + (\cos \theta + i \sin \theta)^2 + \dots$ is **[EAMCET 2001]**

1) i

2) -1

3) 1

4) -1

Ans: 4

Sol. $1 + \text{cis } \theta + (\text{cis } \theta)^2 + \dots \dots, 10\text{th term} = \text{cis } 9\theta$

$$= \cos 9\theta + i \sin 9\theta$$

at $\theta = \frac{\pi}{6} \Rightarrow 10\text{th term} = -i$

7 $\frac{\sin 5\theta}{\sin \theta} =$

[EAMCET 2001]

1) $16 \cos^4 \theta - 12 \cos^2 \theta + 1$

2) $16 \cos^4 \theta + 12 \cos^2 \theta + 1$

3) $16 \cos^4 \theta - 12 \cos^2 \theta - 1$

4) $16 \cos^4 \theta + 12 \cos^2 \theta - 1$

Ans: 1

Sol. $\sin 5\theta = 16 \cos^4 \theta \cdot \sin \theta - 12 \cos^2 \theta \cdot \sin \theta + \sin \theta$

$$\therefore \frac{\sin 5\theta}{\sin \theta} = 16 \cos^4 \theta - 12 \cos^2 \theta + 1$$

