Electrical Capacity

Synopsis

1. Electrical Capacity

i) Electrical capacity of a conductor is its ability to store electric charge.

ii) The potential acquired by a conductor is directly proportional to the charge given to it i.e., $V \propto Q$.

i.e., $Q \propto V$ or $Q = CV$ where the constant of proportionality 'C' is called the electrical capacity of the conductor.

iii) Thus the capacity of a conductor is defined as the ratio of the charge to the potential.

iv) Its SI unit is farad.

v) 1 milli farad (1 mF) = 10^{-3} farad

1 micro farad (1 μF) = 10^{-6} farad

1 pico farad (1 pF) = 10^{-12} farad

vi) The capacity of a spherical conductor in farad is given by $C = 4\pi\varepsilon_0 r$, where r = radius of the conductor.

vii) If we imagine Earth to be a uniform solid sphere then the capacity of earth

$$C = 4\pi\varepsilon_0 R = \frac{6400 \times 10^3}{9 \times 10^9} = 711\mu F \cong 1\ mF$$

2. Parallel plate Capacitor

i) Condenser (usually, a combination of two conductors) is a device by means of which larger amount of charge can be stored at a given potential by increasing its electric capacity.

ii) Capacitance of a capacitor or condenser is the ratio of the charge on either of its plates to the potential difference between them.

iii) Capacity of a parallel plate condenser without medium between the plates

$$C_0 = \frac{\varepsilon_0 A}{d}$$

A = area of each plate; d = distance between the plates

Iv) With a medium of dielectric constant K completely filling the space between the plates

$$C = K\frac{\varepsilon_0 A}{d}$$

v) The **dielectric constant** of a dielectric material is defined as the ratio of the capacity of the parallel plate condenser with the dielectric between the plates to its capacity with air or vacuum between the plates.

$$K = \frac{C}{C_0} = \frac{\text{Capacity of the condenser with dielectric medium between plates}}{\text{Capacity of the same condeser with air as medium between plates}}$$

vi) When a dielectric slab of thickness 't' is introduced between the plates

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{k}} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{k}\right)}$$

vii) In this case the distance of separation decreases by $t\left(1-\frac{1}{k}\right)$ and hence the capacity increases

viii) To restore the capacity to original value the distance of separation is to be increased by $t\left(1-\frac{1}{k}\right)$

ix) a) If a metal slab of thickness t is introduced between the plates $C = \frac{\varepsilon_0 A}{d-t}$ because for metals K is infinity.

b) If a number of dielectric slabs are inserted between the plates, each parallel to plate surface, then equivalent capacity.

$$C = \frac{\epsilon_0 A}{d - t_1\left(1-\frac{1}{K_1}\right) - t_2\left(1-\frac{1}{K_2}\right) \ldots\ldots\ldots\ldots - t_n\left(1-\frac{1}{K_n}\right)}.$$

If those slabs completely fill up the gap between the plates leaving without any air gap $C = \frac{\varepsilon_0 A}{\left(\frac{t_1}{K_1} + \frac{t_2}{K_2} + \ldots\ldots\ldots + \frac{t_n}{K_n}\right)}$.

x) In a parallel plate capacitor, the electric field at the edges is not uniform and that field is called as the **fringing field**.

xi) Electric field between the plates is uniform electric intensity

$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} = \frac{Q}{Cd}$. Here σ is the surface charge density on the plates = Q/A.

xii) Potential difference between the plates $V = E.d = \frac{Q}{\varepsilon_0 A}.d$

xiii) Force on each plate $F = \frac{1}{2}EQ = \frac{1}{2}\frac{Q^2}{Cd} = \frac{1}{2}\frac{CV^2}{d} = \frac{1}{2}\frac{Q^2}{\varepsilon_0 A} = \frac{1}{2}\varepsilon_0 AE^2$

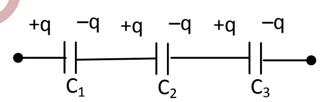
xiv) Energy stored per unit volume of the medium = $\frac{1}{2}\varepsilon_0 E^2$

3. Combination of Condensers

i) When condensers are connected in series

1) All plates have the same charge in magnitude

2) Potential differences between the plates are different



3) $V_1 : V_2 : V_3 = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$

4) Equivalent capacity is C then, $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

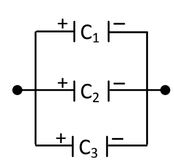
5) The equivalent capacity is less than the least individual capacity

6) Energies of the condensers $E_1 : E_2 : E_3$

$= \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$

7) Total energy of the combination$=E_1+E_2+E_3$.

ii) When condensers are connected in parallel



1) P.D. across each condenser is same

2) Charge of each condenser is different $Q_1 : Q_2 : Q_3 = C_1 : C_2 : C_3$

3) Equivalent capacity of the combination $C = C_1 + C_2 + C_3$

4) The equivalent capacity is greater than the greatest individual capacity

5) Energies of the condensers $E_1 : E_2 : E_3 = C_1 : C_2 : C_3$

6) Total energy of the combination$=E_1+E_2+E_3$

iii) When n identical condensers each of capacity C

1) Combined in series, the effective capacity

$= C_s = C/n$

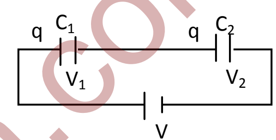
2) Combined in parallel, the effective capacity $C_p = nc$.

3) Ratio of the effective capacities $C_s:C_p = 1: n^2$

iv) **Mixed group:** If there are N capacitors each rated at capacity C and voltage V, by combining those we can obtain effective capacity rated at C^1 and voltage V^1. For this n capacitors are connected in a row and m such rows are connected in parallel.

Then $n = \frac{V^1}{V}$ and $m = \frac{nC^1}{C}$ where $mn = N$

v) If C_p and C_s are the equivalent capacities of two capacitors of capacity C_1 and C_2 in parallel and series respectively then



$C_1 = \frac{1}{2}\left[C_P + \sqrt{C_P^2 - 4C_PC_S}\right]$ and

$C_2 = \frac{1}{2}\left[C_P - \sqrt{C_P^2 - 4C_PC_S}\right]$

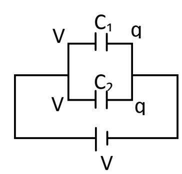
vi) Two capacitors are connected in parallel to a battery as shown in the figure.

i) $V_1 = \frac{VC_2}{C_1 + C_2}$ ii) $V_2 = \frac{VC_1}{C_1 + C_2}$

vii) Two capacitors are connected in parallel to a battery as shown in the figure.

i) $q_1 = \frac{qC_1}{C_1 + C_2}$ ii) $q_2 = \frac{qC_2}{C_1 + C_2}$

viii) If n identical capacitors each of capacity C are connected in a square then



a) The resultant capacity between any two adjacent corners A and B $= \frac{4C}{3}$

b) The resultant capacity between any two opposite corners A and C= C

ix) If n identical capacitors each of capacity C are connected in a polygon then

a) The resultant capacity between any two adjacent corners $= \frac{nC}{n-1}$

b) The resultant capacity between any two opposite corners $= \frac{4C}{n}$

x) a) If n identical capacitors are given then they can be connected in 2^{n-1} different ways by taking all the condensers at a time (n > 2).

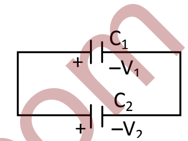
b) In n different capacitors are given then they can be connected in 2^n different ways by taking all the condensers at a time.

xi) In a parallel plate capacitor, if n similar plates at equal distance d are arranged such that alternate plates are connected together, the capacitance (C) of the arrangement is $\frac{(n-1)\varepsilon_0 A}{d}$ for air or vacuum and it becomes $\frac{(n-1)\varepsilon_0 AK}{d}$ in a dielectric medium of dielectric constant K.

4. Energy of capacitor

i) The electrostatic energy stored in a charged capacitor is equal to

$V = \frac{Q^2}{2C}$ or $\frac{CV^2}{2}$ or $\frac{QV}{2}$.



ii) This energy is stored in the uniform electric field that is present between the plates of the capacitor.

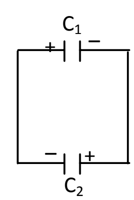
5. Combination of charged capacitors

i) If two condensers of capacities C_1 and C_2 are charged to potentials V_1 and V_2 respectively and are joined in parallel (+ve plate connected to +ve plate), then the common potential

$$V \; \frac{Q_1+Q_2}{C_1+C_2} = \frac{C_1V_1+C_2V_2}{C_1+C_2}.$$

ii) The loss of energy in this process (manifested as heat) is given by

$$U = \frac{C_1C_2}{2(C_1+C_2)}(V_1-V_2)^2.$$



iii) When two condensers of capacities C_1 and C_2 charged to potentials V_1 and V_2 are connected anti-parallel (+ve plate connected to –ve plate) as shown in the figure.

a) Common potential $V = \frac{Q_1-Q_2}{C_1+C_2} \Rightarrow \frac{C_1V_1-C_2V_2}{C_1+C_2}$

b) Loss of energy

$$= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}(C_1+C_2)V^2 \qquad = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1+V_2)^2$$

c) Loss of energy is more in this case compared with previous case.

6. Capacitance of spherical condenser

a) Capacitance of single isolated sphere $= 4\pi\varepsilon_0 R$ where R is its radius.

b) In two concentric spheres (outer radius a and inner radius b)

i) When the inner is charged and the outer is earthed, then

$$C = \frac{4\pi\varepsilon_0\varepsilon_r ab}{a-b} = 4\pi\varepsilon_0 \cdot \frac{Kab}{(a-b)}$$

ii) When the inner sphere is earthed, then

$$C = \frac{4\pi\varepsilon_0\varepsilon_r a^2}{a-b} = \frac{4\pi\varepsilon_0 Ka^2}{a-b}$$

7. Introduction of dielectric in a charged capacitor

A dielectric slab (K) is introduced between the plates of the capacitor.

S.No	Physical quantity	With battery permanently connected	With battery disconnected
1. 2. 3. 4. 5.	Capacity Charge P.D. Electric Intensity	K time increases K times increases Remains constant Remains constant K times increases	K times increases Remains constant K times decreases K times decreases K times decreases

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8. The distance between the plates of condenser is increased by n times.

S.No.	Physical quantity	With battery permanently connected	With battery disconnected
1.	Capacity	n time decreases	n times decreases
2.	Charge	n times decreases	Remains constant
3.	P.D.	Remains constant	n times increases
4.	Electric Intensity	n time decreases	Remain constant
5.	Energy stored in condenser	n times decreases	n times increases

9. Combination of charged spherical drops

When 'n' identical charged small spherical drops are combined to form a big drop

S.no.	Quantity	For each charged small drop	For the big drop
a.	Radius	r	$R = n^{1/3} r$
b.	Charge	q	$Q = n \times q$
c.	Capacity	C	$C^1 = n^{1/3} \times C$
d.	Potential	V	$V^1 = n^{2/3} \times V$
e.	Energy	ν	$\nu^1 = n^{5/3}\, \nu$