Gauss Law and Applications

1. **Statement:** The total normal electric flux ϕ_e over a closed surface is $1/\varepsilon_0$ times the total charge Q enclosed within the surface.

$$\phi_e = \left(\frac{1}{\varepsilon_0}\right)Q$$

2. Gauss Law is applicable for any distribution of charges and any type of closed surface, but it is easy to solve the problem of high symmetry.

3. At any point over the spherical Gaussian surface, net electric field is the vector sum of electric fields due to $+q_1$, $-q_1$ and q_2.

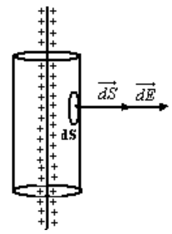
4. **Applications of Gauss Theorem**

a) **Electric field at a point due to a line charge**

A thin straight wire over which 'q' amount of charge be uniformly distributed. l be the linear charge density i.e, charge present per unit length of the wire.

$$E = \frac{q}{2\pi \epsilon_0 rl}$$

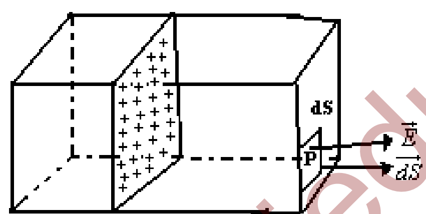
$$E = \frac{\lambda}{2\pi \epsilon_0 r}$$



This implies electric field at a point due to a line charge is inversely proportional to the distance of the point from the line charge.

b) **Electric field intensity at a point due to a thin infinite charged sheet**

'q' amount of charge be uniformly distributed over the sheet. Charge present per unit surface area of the sheet is σ.



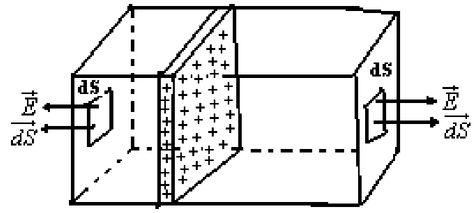
$$E = \frac{q}{2A \in_0}$$

$$E = \frac{q}{2 \in_0} \text{ where } \sigma = \frac{q}{A}$$

E is independent of the distance of the point from the charged sheet.

c) **Electric field intensity at a point due to a thick infinite charged sheet**

'q' amount of charge be uniformly distributed over the sheet. Charge present per unit surface area of the sheet be σ.



$$E = \frac{q}{A \in_0} = \frac{\sigma}{\in_0}$$

Electric field at a point due to a thick charged sheet is twice that produced by the thin charged sheet of same charge density.

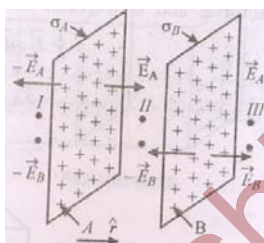
d) **Electric intensity due to two thin parallel charged sheets**

Two charged sheets A and B having uniform charge densities σ_A and σ_B respectively.

In region I

$$E = \frac{1}{2 \in_0}(\sigma_A + \sigma_B)$$

In region II



$$E_{II} = \frac{1}{2 \in_0}(\sigma_A - \sigma_B)$$

In region III

$$E_{III} = \frac{1}{2 \in_0}(\sigma_A + \sigma_B)$$

e) **Electric field due to two oppositely charged parallel thin sheets**

$$E_I = -\frac{1}{2 \in_0}[\sigma + (-\sigma)] = 0$$

$$E_{II} = \frac{1}{2 \in_0}[\sigma - (-\sigma)] = \frac{\sigma}{\in_0}$$

$$E_{III} = \frac{1}{2\in_0}(\sigma - \sigma) = 0$$

f) **Electric field due to a charged Spherical shell**

'q' amount of charge be uniformly distributed over a spherical shell of radius 'R'

$\sigma =$ Surface charge density, $\sigma = \frac{q}{4\pi R^2}$

i) **When point 'P', lies outside the shell**

$$E = \frac{1}{4\pi\in_0} \times \frac{q}{r^2}$$

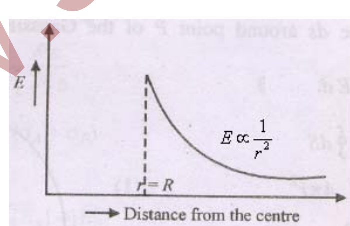
This is the same expression as obtained for electric field at a point due to a point charge. Hence a charged spherical shell behaves as a point charge concentrated at the centre of it.

$$E = \frac{1}{4\pi\in_0}\frac{\sigma.4\pi R^2}{r^2} \quad \because \sigma = \frac{q}{4\pi r^2} \qquad E = \frac{\sigma.R^2}{\in_0 r^2}$$

ii) **When point 'P', lies on the shell:** $E = \frac{\sigma}{\in_0}$

iii) **When Point 'P' lies inside the shell**

E = 0



The electric intensity at any point due to a charged conducting solid sphere is same as that of a charged conducting spherical shell.

g) **Electric Potential (V) due to a spherical charged conducting shell (Hollow sphere)**

q

R r

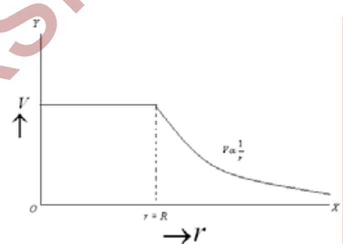
r

i) When point (P_3) lies outside the sphere $(r > R)$, the electric potential, $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

ii) When point (P_2) lies on the surface $(r = R)$, $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$

iii) When point (P_1) lies inside the surface $(r < R)$, $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$

Note: The electric potential at any point inside the sphere is same and is equal to that on the surface.



Note: The electric potential at any point due to a charged conducting sphere is same as that of a charged conducting spherical shell.