Dynamics – Collisions

1. Newton's third law explains the law of conservation of linear momentum

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Where

m_1, m_2 - Masses of the colliding bodies.

u_1, u_2 - Velocities of the bodies before collision.

v_1, v_2 - Velocities of the bodies after collision.

2. Recoil of a Gun: If a bullet of mass 'm' travelling with a muzzle velocity, is fired from a rifle of mass 'M', then

a) Velocity of recoil of the gun is V = mv/M

b) K.E of the bullet is greater than the K.E of the rifle.

c) $\frac{KE_b}{KE_r} = \frac{M}{m} = \frac{v}{V}$

d) When a gun of mass 'M' fire a bullet of mass 'm' releasing a total energy 'E'.

e) Energy of bullet $E_b = \frac{E.M}{M+m}$

f) Energy of gun $E_G = \frac{E.m}{M+m}$

3. When a moving shell explodes, its total (vector sum) momentum remains constant but its total kinetic energy increases.

4. If the velocities of colliding bodies before and after collision are confined to a straight line, it is called head on collision or one dimensional collision.

5. Elastic collisions

1. Both kinetic energy and linear momentum are conserved.
2. Total energy is constant.
3. Bodies will not be deformed.
4. The temperature of the system does not change.

e.g. Collisions between ivory balls, molecular, atomic and nuclear collisions.

6. Perfect elastic collisions

a) When m_1, m_2 are moving with velocities u_1, u_2 and v_1, v_2 before and after collisions, then

a) $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$

b) $v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$

b) Two bodies of equal masses suffering one dimensional elastic collision, exchange their velocities after collision. i.e., if $m_1 = m_2$ then $v_1 = u_2$ and $v_2 = u_1$.

c) If a body suffers an elastic collision with another body of the same mass at rest, the first is stopped dead, whereas the second moves with the velocity of the first.

i.e. if $m_1 = m_2$ and $u_2 = 0$ then $v_1 = 0$; $v_2 = u_1$.

d) When a very light body strikes another very massive one at rest, the velocity of the lighter body is almost reversed and the massive body remains at rest. i.e., if $m_2 >> m_1$ and $u_2 = 0$, then $v_1 \simeq u_1$ and $v_2 = 0$.

e) When a massive body strikes a lighter one at rest, the velocity of the massive body remains practically unaffected where as the lighter one begins to move with a velocity nearly double as much as that of the massive one. i.e., if $m_1 >> m_2$ and $u_2 = 0$, then $v_1 = u_1$ and $v_2 = 2u_1$.

7. A body of mass m_1 collides head on with another body of mass m_2 at rest. The collision is perfectly elastic. Then

a) Fraction of kinetic energy lost by the first body is $\frac{4m_1m_2}{(m_1+m_2)^2}$.

b) Fraction of kinetic energy retained by first body is $\left(\frac{m_1-m_2}{m_1+m_2}\right)^2$.

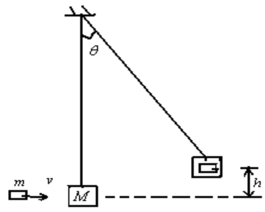
8. Inelastic collision

1. Linear momentum is conserved.
2. Kinetic energy is not conserved.
3. Total energy is conserved.
4. Temperature changes.
5. The bodies may be deformed.
6. The bodies may stick together and move with a common velocity after collision
8. Two bodies collide in one dimension. The collision is perfectly inelastic, then $m_1u_1 + m_2u_2 = (m_1 + m_2) V$
9. Common velocity after collision $v = \frac{m_1u_1+m_2u_2}{(m_1+m_2)}$

10. Total loss of kinetic energy in perfect inelastic collision

$$= \frac{1}{2} \cdot \frac{m_1 m_2 (u_1 - u_2)^2}{m_1 + m_2} = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (u_1 - u_2)^2$$

9. Ballistic Pendulum: A block of mass M is suspended by a light string. A bullet of mass m moving horizontally with a velocity 'v' strikes the block and gets embedded in it. The block and the bullet rise to a height h. Then



a) mv = (M+m)V

b) V = $\sqrt{2gh}$

c) mv = (M+m) $\sqrt{2gh}$

d) v = $\frac{(M+m)}{m}\sqrt{2gh}$

e) If the string of the ballistic pendulum makes an angle θ with vertical after impact and the length of the string is *l* (when θ≤ 90°) $v = \frac{M+m}{m}\sqrt{2gl(1-\cos\theta)}$

f) If the ballistic pendulum just completes a circle in the plane, velocity of the bullet $v = \frac{M+m}{m}\sqrt{5gl}$

10. Co-efficient of restitution (e): The co-efficient of restitution between two bodies in a collision is defined as the ratio of the relative velocity of separation after collision to the relative velocity of their approach before their collision.

a) $e = \frac{\text{relative velocity of separation}}{\text{relative velocity of approach}} = \frac{v_2 - v_1}{u_1 - u_2}$

e = 1 for a perfect elastic collision

e = 0 for a perfect inelastic collision

For any other collision 'e' lies between 0 and 1

b) Loss of K.E. = $\frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(u_1 - u_2)^2 (1 - e^2)$

c) A body falls from a height of h onto a surface of coefficient of restitution 'e'. If it repeatedly bounces for several times, then

i. A body is dropped from a height 'h' rebounds to a height $'h_1'$, then

$$e = \sqrt{\frac{h_1}{h}}$$

ii. For nth rebounce, $h_n = e^{2n} h$

d) A body dropped from a certain height strikes the ground with a velocity 'v' in a time t and rebounds with a velocity v_1 in a time t_1, then

i. $e = \frac{v_1}{v}$ And $e = \frac{t_1}{t}$

ii. After n^{th} bounce, $v_n = e^n v$ and $t_n = e^n t$

iii. Total time taken before it stops rebounding is $T = \sqrt{\frac{2h}{g}}\left[\frac{1+e}{1-e}\right]$

iv. Total displacement of the body before it stops rebounding is h.

v. Total distance traveled before it stops rebounding $s = h\left[\frac{1+e^2}{1-e^2}\right]$

vi. Average velocity before it stops rebounding is $\langle Vel \rangle = \sqrt{\frac{gh}{2}}\left[\frac{1-e}{1+e}\right]$

vii. Average Speed before it stops rebounding $\langle Speed \rangle = \sqrt{\frac{gh}{2}}\left[\frac{1+e^2}{(1+e)^2}\right]$

viii. The time elapses from the moment it is dropped to the second impact with the floor,

$$t = \sqrt{\frac{2h}{g}}(1+2e)$$

11. A ball falls vertically onto a floor with a momentum of P and bounces repeatedly. The total momentum imported by the ball on the floor is given by $\Delta P == p\left(\frac{1+e}{1-e}\right)$

12. If a body is vertically projected up with a velocity u ,

a) The total space covered up to the instant of nth bounce is $H = \frac{u^2}{g}\left[\frac{1-e^{2n}}{1-e^2}\right]$

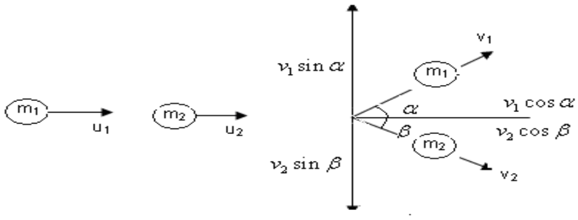
b) Total time taken upto the instant of nth bounce $T = \frac{2u}{g}\left[\frac{1-e^n}{1-e}\right]$

13. A body is projected with a velocity 'u' making an angle 'θ' with horizontal. It makes number of bounces before coming to rest.

a. The total time taken before it stops rebounding is $T = \frac{2u\sin\theta}{g[1-e]}$

b. Total distance traveled before it stops rebounding. $R = \frac{u^2 \sin 2\theta}{g\left(1 - e^2\right)}$

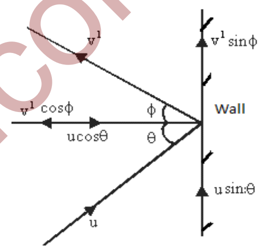
14. Oblique collision: After collision if the bodies move in directions making angle α *and* β with the initial direction of motion, then



$$m_1 u_1 + m_2 u_2 = m_1 v_1 \cos\alpha + m_2 v_2 \cos\beta$$

$$0 = m_1 v_1 \sin\alpha - m_2 v_2 \sin\beta$$

15. A ball strikes a wall with a velocity v at an angle of incidence θ and bounces at an angle of reflection ϕ with a velocity v^1, then



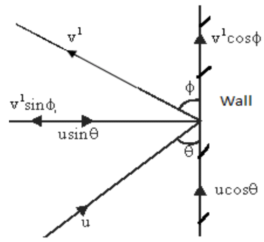
Along the wall, $u \sin\theta = v' \sin\phi$

Perpendicular to the wall, $e = \frac{v_2}{v_1} = \frac{v' \cos\phi}{u \cos\theta}$

$$eu\cos\theta = v'\cos\phi$$

$$\tan\phi = \frac{\tan\theta}{e} \quad \text{and} \quad v' = u\sqrt{\sin^2\theta + e^2\cos^2\theta}$$

16. A ball strikes a wall with a velocity v at an angle θ with the wall and bounces with a velocity v^1 at an angle ϕ with the wall, then



Along the wall, $v^1 \cos\phi = u\cos\theta$

Perpendicular to the wall, $e = \frac{v_2}{v_1} = \frac{v^1 \sin\phi}{u \sin\theta}$

$$v^1 \sin\phi = eu\sin\theta$$