Elasticity

1. **Elasticity:** The property of a body by virtue of which it regains its original size and shape immediately after the removal of deforming forces is called elasticity.
2. **Elastic body:** A body which shows elastic behavior is called elastic body. There are no perfect elastic or plastic bodies in nature. E.g. steel, rubber. Quartz is very nearly perfectly elastic body.
3. **Plastic body:** A body which does not show elastic behavior is called plastic body. E.g. putty, clay, mud, wax, lead, dough, chewing gum, butter wax etc.
4. **Deforming force:** A force which changes or tries to change the shape or size of the body without moving it as a whole is called deforming force.
5. **Restoring force:** A force which is developed inside the body and which tries to regain the original shape or size of the body is called restoring force.
6. A body in which it is more difficult to produce strain is more elastic.

   a) Steel is more elastic than rubber.

   b) Water is more elastic than air.

   c) Springs are made of steel but not of copper because steel is more elastic than copper.
7. **Factors Effecting Elasticity**

   a. Elasticity decreases with the increase of temperature but for nickel and invar steel temperature has no effect on elasticity.

   b. Addition of impurity to metal may increase or decrease the elasticity

   c. Hammering and rolling increases the elasticity.

   d. Annealing decreases the elasticity.
8. **Stress:** The restoring force developed per unit area of cross-section of the deformed body is called stress.

   $$\text{Stress} = \frac{\text{Restoring force}}{\text{Cross}-\text{sectional area}} = \frac{F}{A}$$

   $$\text{Unit} = \frac{N}{m^2} \quad or \quad \text{pascal}$$

   Dimensional formula: $M^1L^{-1}T^{-2}$

9. Stress is of three types

i) **Longitudinal stress:** If the restoring forces are perpendicular to the area of cross-section and are along the length of the wire, the stress is called longitudinal stress.

ii) **Tangential stress or shearing stress:** If the restoring forces are parallel to the surface, the stress is called shearing stress. Here the body under goes a change in shape but not volume. Shearing strain is applicable only for solids.

10. Bulk stress or volume stress: If a body is subjected to equal forces normally on all the faces, the stress involved is called bulk stress. Here the body under goes a change in volume but not shape. Bulk strain is applicable only for fluids.

11. Strain: The ratio of change in dimension of the body to its original dimension is called strain. Strain has no unit and no DF.

a) Longitudinal strain $= \frac{\text{change in length}}{\text{original length}} = \frac{e}{l}$

b) Shearing strain = θ= $\frac{\textit{Lateral}\text{ displacement between two layers}}{\text{Perpendicular distance between the two layers}}$

c) Bulk strain= $\frac{\text{change in volume}}{\text{original volume}} = \frac{\Delta v}{v}$

c) Shearing strain = 2 x longitudinal strain

d) Bulk strain = 3 x longitudinal strain

e) Longitudinal strain: shearing strain: bulk strain = 1:2:3

12. Hooke's law: Within the elastic limit of a body, stress is directly proportional to strain.

$\frac{\text{stress}}{\text{strain}} = E = \text{constant}$

Unit of E: $\frac{\text{newton}}{m^2}$ or Pascal

13. Types of moduli of elasticity: There are three moduli of elasticity.

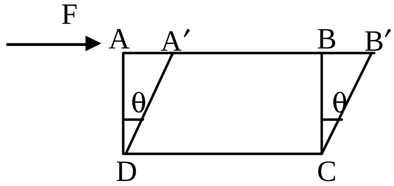
a) Young's modulus: Young's modulus is the ratio of longitudinal stress to longitudinal strain within the elastic limit of a body.

$$Y = \frac{\textit{longitudinal}\text{ stress}}{\textit{longitudinal}\text{ strain}} = \frac{Fl}{Ae}$$

When a mass "M" is attached to the lower end of a wire,

$$Y = \frac{gl}{\pi r^2}\left(\frac{M}{e}\right)$$

b) **Rigidity modulus:** Rigidity modulus is the ratio between shearing stress and shearing strain within the elastic limit of a body.



$$n = \frac{shearing\ \text{stress}}{shearing\ \text{strain}} = \frac{F}{A.\tan\theta}$$

$$n = \frac{F}{A} \times \left(\frac{AD}{AA'}\right)$$

c) **Bulk modulus:** Bulk modulus is the ratio between volume stress and volume strain within the elastic limit of a body.

$$K = \frac{\text{volume stress}}{\text{volume strain}} = \frac{\frac{F}{A}}{-\left(\frac{\Delta v}{v}\right)}$$ (Negative sign indicates the decrease in volume)

Or $K = \frac{Pv}{\Delta v}$

14. Poisson's ratio $(\sigma) = \frac{\text{Lateral contraction strain}}{\text{Longitudinal elongation strain}} = -\frac{\Delta r / r}{\Delta l / l}$

i) Poisson's ratio has no unit and has no dimensions.

ii) Theoretical limits of $\sigma = -1$ to 0.5.

iii) Practical limit of $\sigma = 0$ to 0.5

iv) When the volume of a wire is constant, then $\sigma = 0.5$.

v) A cylindrical rod is made of material of poisons ratio σ. When it is stretched, the fractional change in its volume is $\frac{\Delta v}{v} = \frac{\Delta l}{l}(1-2\sigma)$ (or) $\frac{\Delta v}{v} = \frac{\Delta r}{r}\left(2 - \frac{l}{\sigma}\right)$.

15. Relation among elastic constants Y, η, K, σ :

i) $\frac{9}{Y} = \frac{1}{K} + \frac{3}{\eta}$ ii) $Y = 2\eta(1+\sigma)$

iii) $Y = 3K(\tilde{1}2\sigma)$ iv) $\sigma = \frac{3K - 2\eta}{2(\eta + 3K)}$

16. Linear strain is applicable only for solids.

17. Y is infinity for a perfect elastic material.

18. Y is zero for gases, liquids and perfectly inelastic materials.

19. The reciprocal of bulk modulus $\left(\frac{1}{K}\right)$ is called compressibility(C).

20. The strain produced in a loaded spring is shearing strain.

21. The temporary loss of elasticity of a body due to successive stress and strain is called elastic fatigue.

22. The time delay in regaining the original shape (or) size of the body after removing the deforming force is called elastic after effect.

23. Elastic after effect is not exhibited by quartz or phosphor bronze.

24. If l_1 and l_2 are the length of a wire under tension T_1 and T_2, the actual length of the wire = $\frac{l_1T_2 - l_2T_1}{T_2 - T_1}$

25. .Elongation produced in a wire due to its own weight $e = \frac{l^2dg}{2Y}$

26. Length of the wire which breaks under its own weight $L = \frac{P}{dg}$ Where P is the breaking stress

27. Isothermal elasticity = pressure of the gas

28. Adiabatic elasticity = $\gamma \times$ pressure of the gas

29. Thermal stress $\left(\frac{F}{A}\right) = Y\alpha t$ (or) $F = YA\alpha t$

Were α the coefficient of linear expansion of the solid and t is the rise in temperature of the solid.

Similarly Pressure

$(P) = K\gamma t$ (or) $K = \frac{P}{\gamma t} = \frac{P}{3\alpha t}$

γ is the coefficient of volume expansion of a solid.

30. Strain energy

Work done in stretching a string is stored as elastic potential energy.

$$W = \frac{1}{2}\text{stress} \times \text{strain} \times \text{volume}$$

$$W = \frac{1}{2}Fe \Rightarrow \frac{W}{V} = \frac{1}{2}Y(strain)^2 \text{ or } W = \frac{1}{2}\frac{(stress)^2}{Y}$$

Also, Potential energy stored in a wire due to twisting= $\frac{1}{2}\tau\theta$

31. Strain energy density

Energy per unit volume $= \frac{\text{work}}{\text{volume}} = \frac{1}{2}$ x stress x strain $= \frac{1}{2}\frac{(stress)^2}{y}$

32. Springs

i) For a spring that obeys Hooke's law, equivalent force constant or spring constant is K $= \frac{YA}{l}$.

ii) If a spring (or a wire) of force constant K is cut into 'n' equal parts, the force constant of each part of the wire is 'nk'.

iii) If a spring (or a wire) of force constant k is cut in the ratio of m:n, $k_m = \frac{(m+n)k}{m}$; $k_n = \frac{(m+n)k}{n}$

iv) Potential energy of a stretched spring $= \frac{1}{2}Fx = \frac{1}{2}Kx^2 = \frac{1}{2}\frac{F^2}{K}$

v) Two springs have force constants K_1 and K_2

a) When they are stretched by the same force and if their elastic energies are E_1 and E_2. $\frac{E_1}{E_2} = \frac{K_2}{K_1}$

b) When they are extended by the same length $\frac{E_1}{E_2} = \frac{K_2}{K_1}$

c) When they are extended till their energies are same, $\frac{F_1}{F_2} = \sqrt{\frac{K_1}{K_2}}$

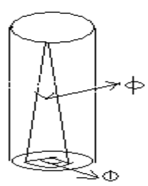
d) The potential energy of a spring increases, whether it is stretched or compressed.

e) Springs in series $K_{eff} = \frac{K_1K_2}{K_1+K_2}$

f) Springs in parallel $K_{eff} = K_1 + K_2$

g) The reciprocal of spring constant is called compliance.

25. If a rod of length l and radius r is fixed at one end and the other end is twisted by an angle θ, then lϕ= rθ. Where ϕ is angle of shear.



The upper end of a cylinder is clamped and a tangential force is applied at the lower end so that the cylinder is twisted through an angle θ shearing strain developed is

$$r\theta = l\Phi \quad \text{or} \quad \Phi = \frac{r\theta}{l}$$

26. Behavior of a wire under the action of a load :

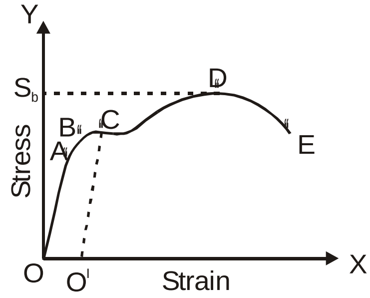
a) A = Proportionality limit

B = Elastic limit

C = Yielding point

D = Breaking point

S_b = Ultimate tensile strength



b) Stress is proportional to strain upto a limit, which is called proportionality limit. A is the limit of proportionality. Upto this limit, Hooke's law is obeyed.

c) The smallest value of stress which produces a permanent change in the body is called elastic limit.

d) If the wire is loaded beyond the elastic limit, a stage is reached where the wire begins to flow with no increase in the load and this point is called yield point.

e) Beyond the yield point, if the load is increased further the extension increases rapidly and the wire becomes narrower and finally breaks. The point at which the wire breaks is called breaking point.

f) A permanent set (OP) is produced in the wire beyond elastic limit.

g) The stress required to reach the breaking point is called breaking stress.

h) If the gap between elastic limit and breaking point (BD) of a metal is large, it is called a ductile metal.

i) If the wire breaks soon after exceeding limit, the metal is said to be brittle. (If the gap BD is small).