Measurements and Errors

Accuracy: Closeness of the measured value to the true value is called accuracy

Precision: Closeness of the measurements done with an instrument to one another called Precision

E.g.: The time period of a second's pendulum is T = 2 sec

Clock A: 2.10 sec, 2.01 sec. 1.98 sec (Accurate)

Clock B: 2.56 sec, 2.57 sec 2.57 sec (Precise)

Error: The difference between the measured value and true value of a physical quantity is called Error

Type of errors: 1) Systematic Errors 2) Random Errors and 3) Gross Errors

1) Systematic Errors: These may be either positive or negative.

a) Constant or Instrumental errors: These are due to i) imperfect design and ii) zero error

b) Imperfection in experimental arrangement: In the calorimeter experiment, the loss of heat due to radiation, the effect on weighing due to buoyancy of air cannot be avoided.

c) Environmental Errors like changes in temperature, pressure wind velocity etc.

d) Personal or Observational errors are due to the improper setting of the apparatus, carelessness in taking observations

2) Random Errors: These are due to fluctuations in temperature, voltage supply etc .

Accurate value can be obtained by taking a number of readings and finding the arithmetic mean of all the readings.

3) Gross Errors: These due to the carelessness of the observer in taking measurements towards the sources of error.

In tangent galvanometer experiment, the coil should be placed in magnetic meridian position and other magnetic materials should be kept away. Neglecting these precaution result in gross errors

No corrections can be applied to these errors. Care should be taken to avoid these errors

Estimation of errors

a) Absolute Errors (Δa) : The magnitude of the difference between the true value of a physical quantity and the individual measured value is called absolute error of that measurement

Absolute error = $|\text{True value} - \text{measured value}|$

Or $|\Delta\alpha_i| = |\alpha_{mean} - \alpha_i|$

Absolute error is always positive. It has the same units as that of the quantity measured

b) **Mean absolute Error** $(|\alpha_{mean}|)$: The arithmetic mean of all the absolute errors is called mean absolute error (or) final absolute error

Mean absolute error

$$\Delta\alpha_{mean} = \frac{|\Delta\alpha_1| + |\Delta\alpha_2| + |\Delta\alpha_3| + ... + |\Delta\alpha_n|}{n} = \frac{1}{n}\sum_{i=1}^{n}|\Delta\alpha_i|$$

Mean absolute error is always positive and has the same units as that of the measured physical quantity.

c) **Relative Error**: The ratio of mean absolute error to the mean value of the quantity measured is called relative error.

Relative error $= \frac{\Delta\alpha_{mean}}{\Delta\alpha_{mean}}$

Relative error has no units.

d) **Percentage Error** (δ_α) : When the relative error is multiplied by 100, it is called percentage error $\delta_\alpha = \left(\frac{\Delta\alpha_{mean}}{\alpha_{mean}} \times 100\right)\%$

Combination of errors

a) **Error of a sum or a difference**

i) If x = a + b

Let Δa *and* Δb be the absolute errors in a and b respectively. Let the error in x be Δx

x = a + b

Maximum possible value of $\Delta x = \Delta a + \Delta b$

Relative error, $\frac{\Delta x}{x} = \frac{\Delta a + \Delta b}{a + b}$

Percentage error, $\frac{\Delta x}{x}\% = \left[\left(\frac{\Delta a + \Delta b}{a + b}\right)100\right]\%$

ii) If x = a – b

Maximum possible value of $\Delta x = \Delta a + \Delta b$

Relative error, $\frac{\Delta x}{x} = \frac{\Delta a + \Delta b}{a - b}$

Percentage error, $\frac{\Delta x}{x}\% = \left[\left(\frac{\Delta a + \Delta b}{a - b}\right)100\right]\%$

b) **Errors of multiplication or Division**

i) If x = a b

Maximum relative error, $\frac{\Delta x}{x} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$

Percentage error, $\frac{\Delta x}{x}\% = \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)100\%$

ii) If $x = \frac{a}{b}$

Maximum relative error $\frac{\Delta x}{x}\% = \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)100\%$

c) Errors of a measured quantity that involves product of powers of observed quantities:

i) If $x = a^n$

Maximum relative error, $\frac{\Delta x}{x}\% = \left(n\frac{\Delta a}{a}100\right)\%$

ii) If $x = \frac{a^p b^q}{c^r}$

Maximum relative error, $\frac{\Delta x}{x} = p\frac{\Delta a}{a} + q\frac{\Delta b}{b} + r\frac{\Delta c}{c}$

Percentage error, $\frac{\Delta x}{x}\% = \left(p\frac{\Delta a}{a} + q\frac{\Delta b}{b} + r\frac{\Delta c}{c}\right)100\%$

Significant figure: Significant figures in a measurement are defined as the number of digits that are known reliably plus the uncertain digit

Rules for determining the number of significant figures

1. All the non-zero digits in a given number are significant without any regard to the location of the decimal point if any

   E.g.: 4205, 42.05, 4.205, 420.5 all have 4 significant digits.

2. All zeros occurring between two non-zero digits are significant without any regard to the location of decimal point if any

   E.g.: 2.002, 20.02, 200.2 all have 4 significant digits.

3. All the zeros to the right of the decimal point but to the left of the first non zero digit are not significant

   E.g.: 0.003 in these number significant digits are 1.

4. All zeros to the right of the last non zero digit in a number after the decimal point are significant

   E.g.: 0.4200 has 4 significant figures

5. All zeros to the right of the last non zero digit in a number having no decimal point are not significant

   E.g.: 4200 has 2 significant figures

   But if the zeros are obtained from actual measurement, then the number of significant figures in 4200 are 4.

Rounding off

1. The preceding digit is raised by one if the immediate insignificant digit to be dropped is more than 5

   E.g.: When 4228 is rounded off to three significant figures, it becomes 4230

2. The preceding digit is to be left unchanged if the immediate insignificant digit to be dropped is less than 5

   E.g.: If 4228 is rounded off to two significant figures it becomes 4200

3. If the immediate insignificant digit to be dropped is 5 then there will be two different cases

   a) If the preceding digit is even, it is to be unchanged and 5 is dropped

   Eg: If 4.728 is to be rounded off to two decimal places, it becomes 4. 72

   b) If the preceding digit is odd, it is to be raised by 1

   E.g.: If 4.7358 is to be rounded off two decimal places it becomes 4.74

Rules for arithmetic operations with significant figures

1. Addition and subtraction:

   For addition and subtraction, the rule in terms of decimal places

   i) After completing addition or subtraction, round off the final result to the least number of decimal places (n)

   Eg 1): Find the value of 2.2 + 5.08 + 3.125 + 5.3755

   Ans: 15.78 is rounded off to 15.8

   Eg (2): Find the value of 44.8 – 21.235

   Ans: 23.565 is rounded off to 23.6

2. **Multiplication and division**

   In multiplication and division, the rule is in terms of significant figures

   i) In a given set of numbers, notice the number with the least number of significant figures (n) and the round off the other number to $(n+1)$ significant figures. Complete the arithmetic operation

   ii) After completing multiplication or division round off the final result to the least number of significant figures (n)

E.g.: (1): Find the value of $1.2 \times 2.54 \times 3.257$

$1.2 \times 2.54 \times 3.26 = 9.93468$

Final result is rounded off to 9.9

Eg: 2) Find $9.27 \div 41$

$\frac{9.27}{41} = 0.2260975$

Final result is rounded off to 0.23